

# PONDEROMOTIVE INSTABILITIES MICROPHONICS RF CONTROL

Jean Delayen

Thomas Jefferson National Accelerator Facility  
Old Dominion University

# Frequency Control

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Energy Gain

$$W = q V \cos \phi$$

Energy gain error

$$\frac{\delta W}{W} = \frac{\delta V}{V} - \delta \phi \tan \phi$$

The fluctuations in cavity field amplitude and phase come mostly from the fluctuations in cavity frequency

Need for fast frequency control

Minimization of rf power requires matching of average cavity frequency to reference frequency

Need for slow frequency tuners

# Some Definitions

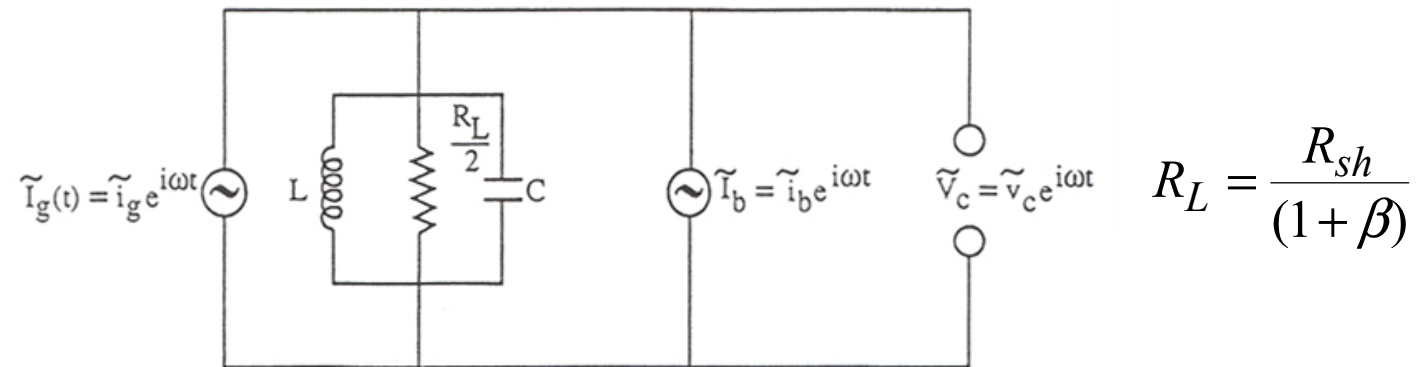
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- Ponderomotive effects: changes in frequency caused by the electromagnetic field (radiation pressure)
  - Static Lorentz detuning (cw operation)
  - Dynamic Lorentz detuning (pulsed operation)
- Microphonics: changes in frequency caused by connections to the external world
  - Vibrations
  - Pressure fluctuations

Note: The two are not completely independent.  
When phase and amplitude feedbacks are active, ponderomotive effects can change the response to external disturbances

# Equivalent Circuit for a Cavity with Beam

- Beam in the rf cavity is represented by a current generator.
- Equivalent circuit:



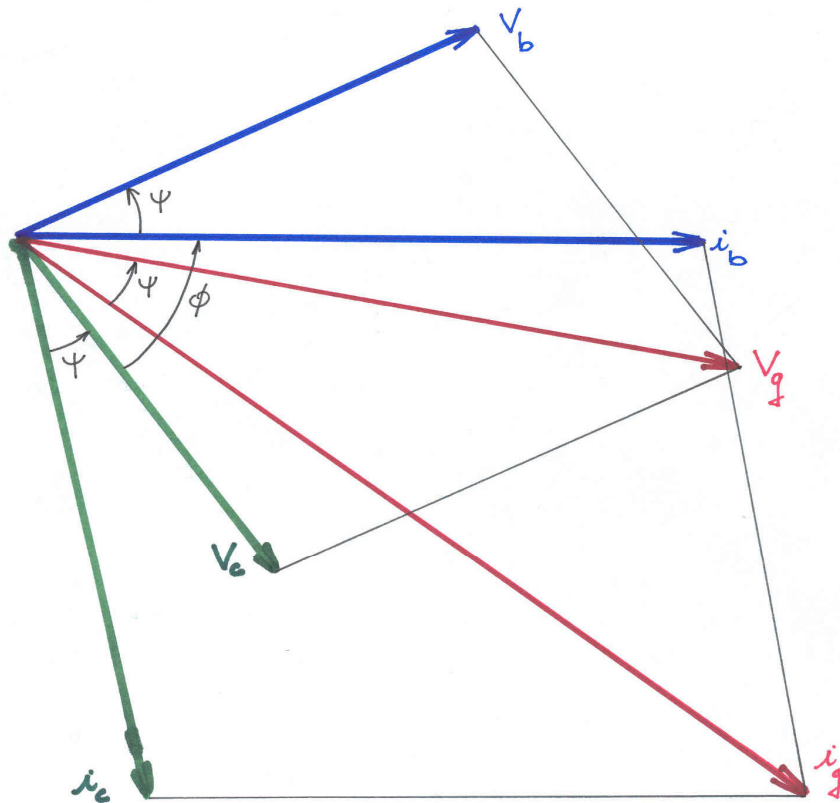
$\tilde{i}_b$  produces  $\tilde{V}_b$  with phase  $\psi$  (detuning angle)

$\tilde{i}_g$  produces  $\tilde{V}_g$  with phase  $\psi$

$$\tilde{V}_c = \tilde{V}_g - \tilde{V}_b$$

$$\tan \psi = -2 \frac{Q_0}{1 + \beta} \frac{\Delta \omega}{\omega_0}$$

# Equivalent Circuit for a Cavity with Beam



$$V_g = (P_g R_{sh})^{1/2} \frac{2\beta^{1/2}}{1+\beta} \cos \psi$$

$$V_b = \frac{i_b R_{sh}}{2(1+\beta)} \cos \psi$$

$$i_b = 2i_0 \frac{\sin \frac{\theta_b}{2}}{\frac{\theta_b}{2}}$$

$i_b$ : beam rf current

$i_0$ : beam dc current

$\theta_b$ : beam bunch length

# Equivalent Circuit for a Cavity with Beam

$$P_g = \frac{V_c^2}{R_{sh}} \frac{1}{4\beta} \left\{ (1 + \beta + b)^2 + [(1 + \beta) \tan \psi - b \tan \phi]^2 \right\}$$

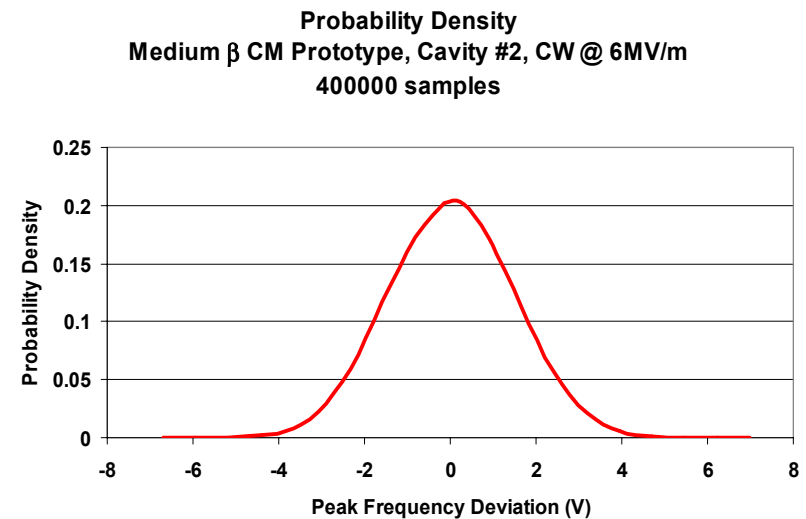
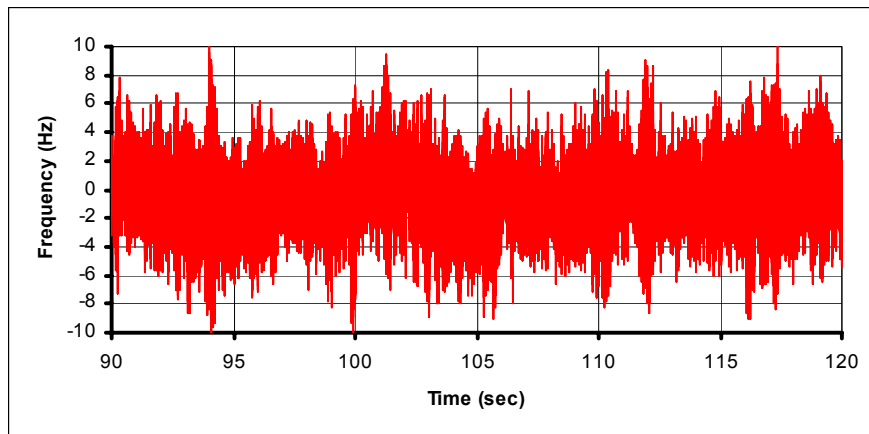
$$b = \frac{\text{Power absorbed by the beam}}{\text{Power dissipated in the cavity}} = \frac{R_{sh} i_0 \cos \phi}{V_c}$$

Minimize  $P_g$  :

$$(1 + \beta_{opt}) \tan \psi_{opt} = b \tan \phi$$
$$\beta_{opt} = |1 + b|$$
$$P_g^{opt} = \frac{V_c^2}{R_{sh}} \frac{|1 + b| + (1 + b)}{2}$$

# Cavity with Beam and Microphonics

- The detuning is now  $\tan \psi = -2Q_L \frac{\delta\omega_0 \pm \delta\omega_m}{\omega_0}$   $\tan \psi_0 = -2Q_L \frac{\delta\omega_0}{\omega_0}$   
where  $\delta\omega_0$  is the static detuning (controllable)  
and  $\delta\omega_m$  is the random dynamic detuning (uncontrollable)



# $Q_{\text{ext}}$ Optimization with Microphonics

- Condition for optimum coupling:

$$\beta_{opt} = \sqrt{(b+1)^2 + \left(2Q_0 \frac{\delta\omega_m}{\omega_0}\right)^2}$$

and

$$P_g^{opt} = \frac{V_c^2}{2R_{sh}} \left[ (b+1) + \sqrt{(b+1)^2 + \left(2Q_0 \frac{\delta\omega_m}{\omega_0}\right)^2} \right]$$

- In the absence of beam (b=0):

$$\beta_{opt} = \sqrt{1 + \left(2Q_0 \frac{\delta\omega_m}{\omega_0}\right)^2}$$

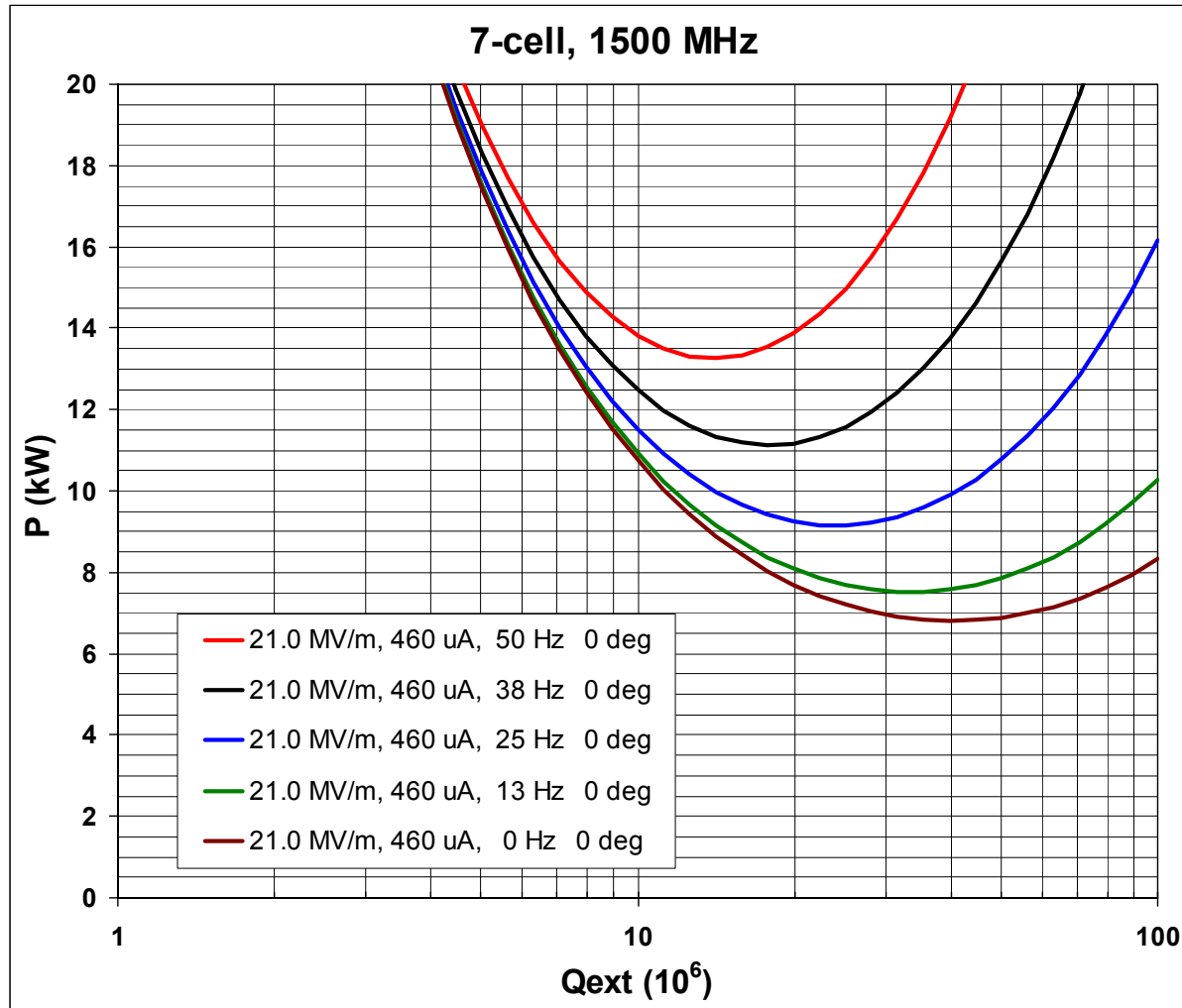
and

$$P_g^{opt} = \frac{V_c^2}{2R_{sh}} \left[ 1 + \sqrt{1 + \left(2Q_0 \frac{\delta\omega_m}{\omega_0}\right)^2} \right]$$

$\simeq U \delta\omega_m$  If  $\delta\omega_m$  is very large

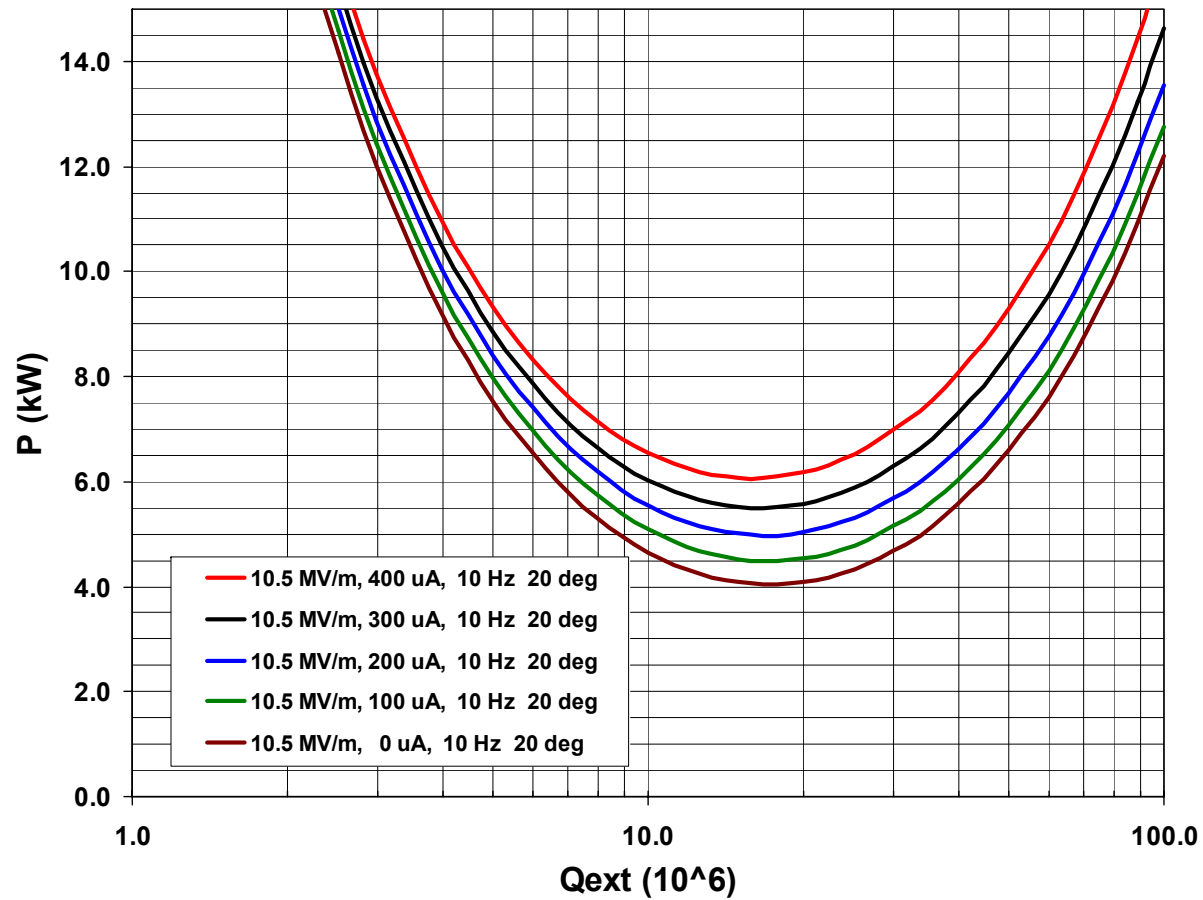


# Example



# Example

3-spoke, 345 MHz,  $\beta=0.62$



# Lorentz Detuning

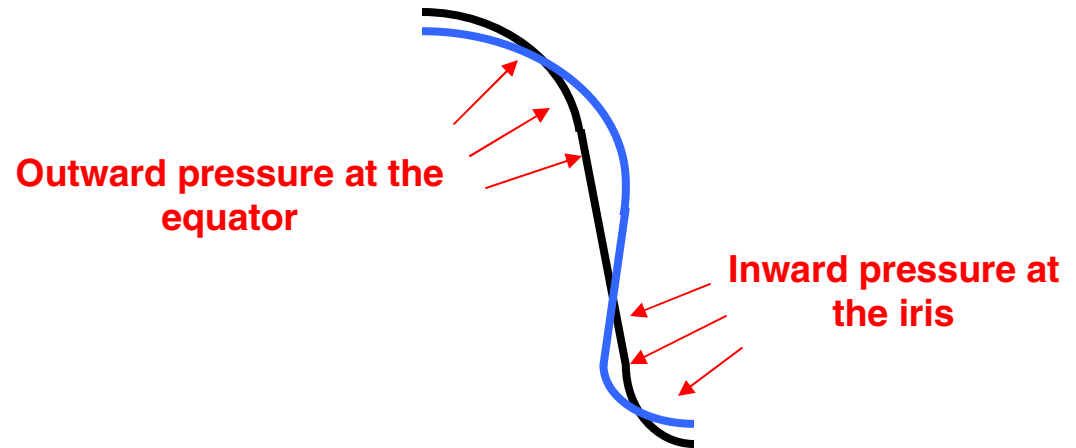
- RF power produces radiation

pressure:  $P = (\mu_0 H^2 - \epsilon_0 E^2)/4$

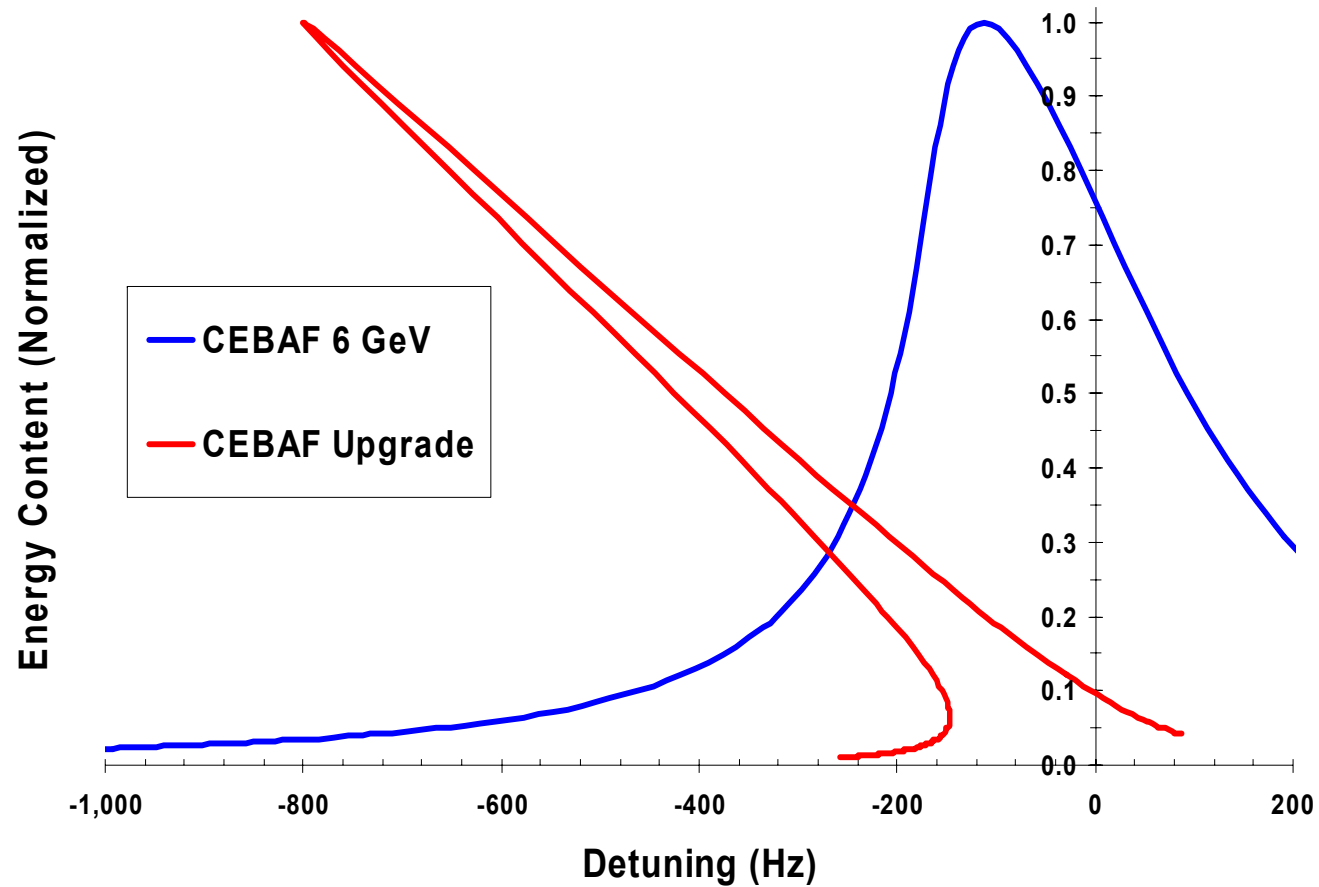
- Deformation produce a frequency shift:

$$\Delta f = -k_L * E_{acc}^2$$

Pressure deforms the cavity wall:

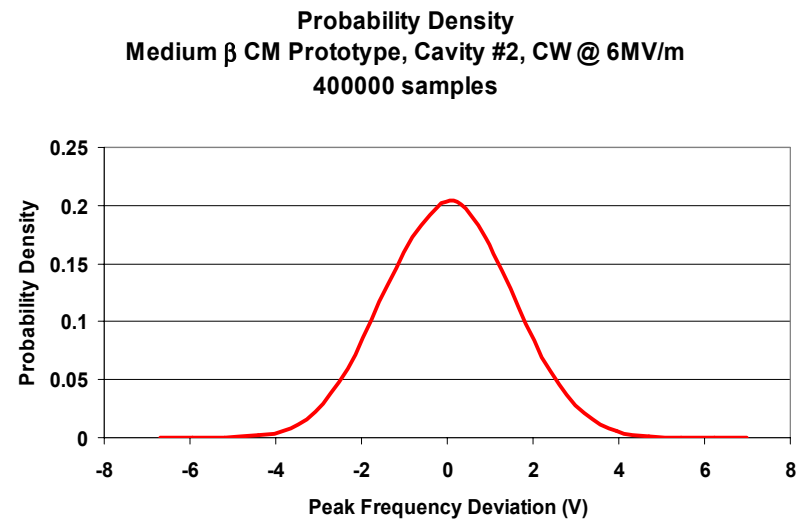
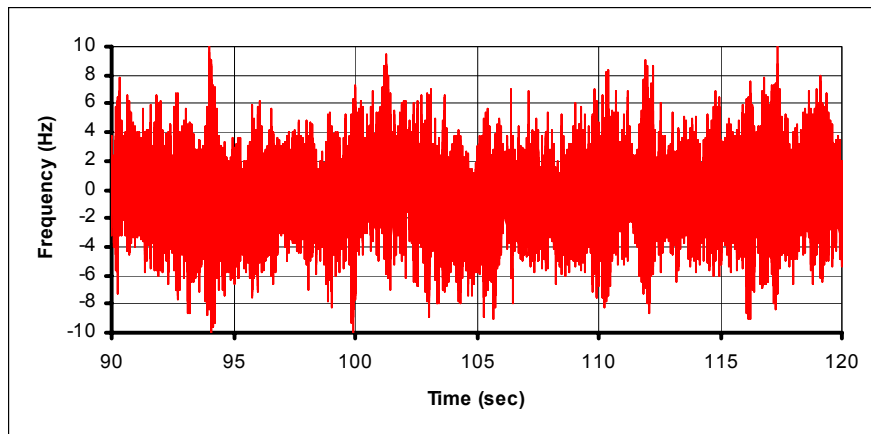


# Lorentz Detuning



# Microphonics

- **Total detuning**  $\delta\omega_0 + \delta\omega_m$   
where  $\delta\omega_0$  is the static detuning (controllable)  
and  $\delta\omega_m$  is the random dynamic detuning (uncontrollable)



# Ponderomotive Effects

- **Adiabatic theorem applied to harmonic oscillators (Boltzmann-Ehrenfest)**

If  $\varepsilon = \frac{1}{\omega^2} \frac{d\omega}{dt} \ll 1$ , then  $\frac{U}{\omega}$  is an adiabatic invariant to all orders

$$\Delta \left( \frac{U}{\omega} \right) / \left( \frac{U}{\omega} \right) \sim o(e^{-d/\varepsilon}) \Rightarrow \boxed{\frac{\Delta \omega}{\omega} = \frac{\Delta U}{U}} \quad (\text{Slater})$$

Quantum mechanical picture: the number of photons is constant:  $U = N\hbar\omega$

$$U = \int_V dV \left[ \frac{\mu_0}{4} H^2(\vec{r}) + \frac{\varepsilon_0}{4} E^2(\vec{r}) \right] \quad (\text{energy content})$$

$$\Delta U = - \int_S dS \vec{n}(\vec{r}) \cdot \vec{\xi}(\vec{r}) \left[ \frac{\mu_0}{4} H^2(\vec{r}) - \frac{\varepsilon_0}{4} E^2(\vec{r}) \right] \quad (\text{work done by radiation pressure})$$

# Ponderomotive Effects

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$$\frac{\Delta\omega}{\omega} = - \frac{\int_S dS \vec{n}(\vec{r}) \cdot \vec{\xi}(\vec{r}) \left[ \frac{\mu_0}{4} H^2(\vec{r}) - \frac{\epsilon_0}{4} E^2(\vec{r}) \right]}{\int_V dV \left[ \frac{\mu_0}{4} H^2(\vec{r}) + \frac{\epsilon_0}{4} E^2(\vec{r}) \right]}$$

Expand wall displacements and forces in normal modes of vibration  $\phi_\mu(\vec{r})$  of the resonator

$$\int_S dS \phi_\mu(\vec{r}) \phi_\nu(\vec{r}) = \delta_{\mu\nu}$$

$$\xi(\vec{r}) = \sum_{\mu} q_{\mu} \phi_{\mu}(\vec{r}) \qquad q_{\mu} = \int_S \xi(\vec{r}) \phi_{\mu}(\vec{r}) dS$$

$$F(\vec{r}) = \sum_{\mu} F_{\mu} \phi_{\mu}(\vec{r}) \qquad F_{\mu} = \int_S F(\vec{r}) \phi_{\mu}(\vec{r}) dS$$

# Ponderomotive Effects

Equation of motion of mechanical mode  $\mu$

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{q}_\mu} - \frac{\partial L}{\partial q_\mu} + \frac{\partial \Phi}{\partial \dot{q}_\mu} = F_\mu \quad L = T - U \quad (\text{Euler-Lagrange})$$

$$U = \frac{1}{2} \sum_{\mu} c_{\mu} q_{\mu}^2 \quad (\text{elastic potential energy}) \quad c_{\mu}: \text{elastic constant}$$

$$T = \frac{1}{2} \sum_{\mu} c_{\mu} \frac{\dot{q}_{\mu}^2}{\Omega_{\mu}^2} \quad (\text{kinetic energy}) \quad \Omega_{\mu}: \text{frequency}$$

$$\Phi = \sum_{\mu} \frac{c_{\mu}}{\tau_{\mu}} \frac{\dot{q}_{\mu}^2}{\Omega_{\mu}^2} \quad (\text{power loss}) \quad \tau_{\mu}: \text{decay time}$$

$$\ddot{q}_{\mu} + \frac{2}{\tau_{\mu}} \dot{q}_{\mu} + \Omega_{\mu}^2 q_{\mu} = \frac{\Omega_{\mu}^2}{c_{\mu}} F_{\mu}$$



# Ponderomotive Effects

The frequency shift  $\Delta\omega_\mu$  caused by the mechanical mode  $\mu$  is proportional to  $q_\mu$

$$\Delta\ddot{\omega}_\mu + \frac{2}{\tau_\mu} \Delta\dot{\omega}_\mu + \Omega_\mu^2 \Delta\omega_\mu = -\frac{\omega_0}{c_\mu} \left( \frac{F_\mu}{U} \right)^2 \Omega_\mu^2 U = -k_\mu \Omega_\mu^2 V^2$$

Total frequency shift:  $\Delta\omega(t) = \sum_\mu \Delta\omega_\mu(t)$

Static frequency shift:  $\Delta\omega_0 = \sum_\mu \Delta\omega_{\mu 0} = -V^2 \sum_\mu k_\mu$

Static Lorentz coefficient:  $k = \sum_\mu k_\mu$

# Ponderomotive Effects – Mechanical Modes

$$\Delta\ddot{\omega}_\mu + \frac{2}{\tau_\mu} \Delta\dot{\omega}_\mu + \Omega_\mu^2 \Delta\omega_\mu = -\Omega_\mu^2 k_\mu V_o^2 + \cancel{n(t)}$$

**Fluctuations around steady state:**

$$\Delta\omega_\mu = \Delta\omega_{\mu 0} + \delta\omega_\mu$$

$$V = V_0(1 + \delta v)$$

Linearized equation of motion for mechanical mode:

$$\delta\ddot{\omega}_\mu + \frac{2}{\tau_\mu} \delta\dot{\omega}_\mu + \Omega_\mu^2 \delta\omega_\mu = -2\Omega_\mu^2 k_\mu V_o^2 \delta v$$

The mechanical mode is driven by fluctuations in the electromagnetic mode amplitude.

Variations in the mechanical mode amplitude causes a variation of the electromagnetic mode frequency, which can cause a variation of its amplitude.

→ Closed feedback system between electromagnetic and mechanical modes, that can lead to instabilities.

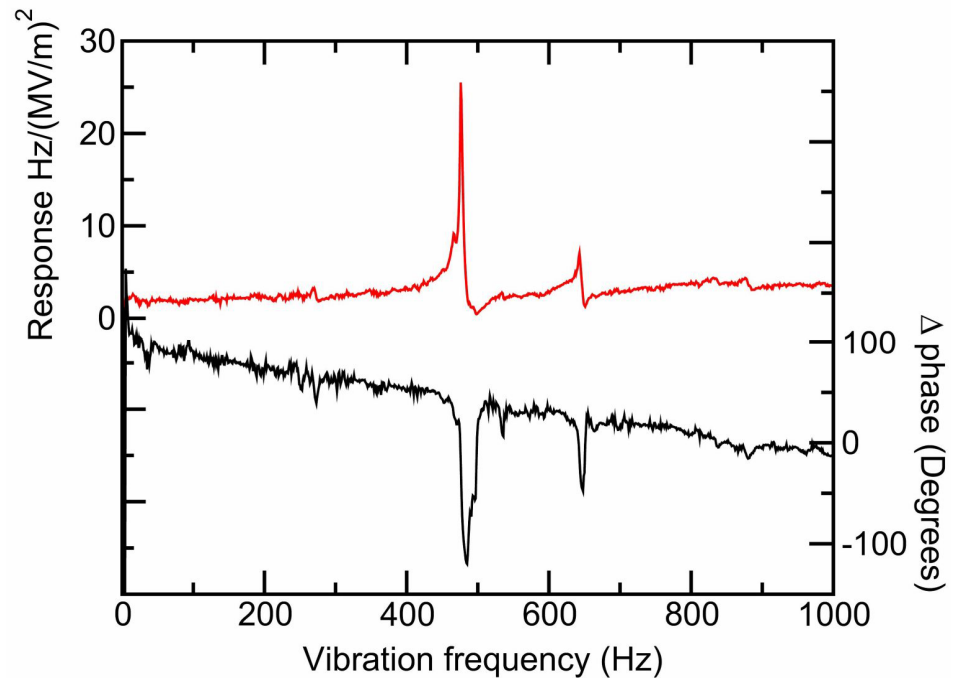
# Lorentz Transfer Function

$$\delta\ddot{\omega}_\mu + \frac{2}{\tau_\mu} \delta\dot{\omega}_\mu + \Omega_\mu^2 \delta\omega_\mu = -2\Omega_\mu^2 k_\mu V_o^2 \delta v$$

$$\delta\omega_\mu(\omega) = \frac{-2\Omega_\mu^2 k_\mu V_o^2}{(\Omega_\mu^2 - \omega^2) + \frac{2}{\tau_\mu} i\omega} \delta v(\omega)$$

TEM-class cavities  
ANL, single-spoke,  
354 MHz,  $\beta=0.4$

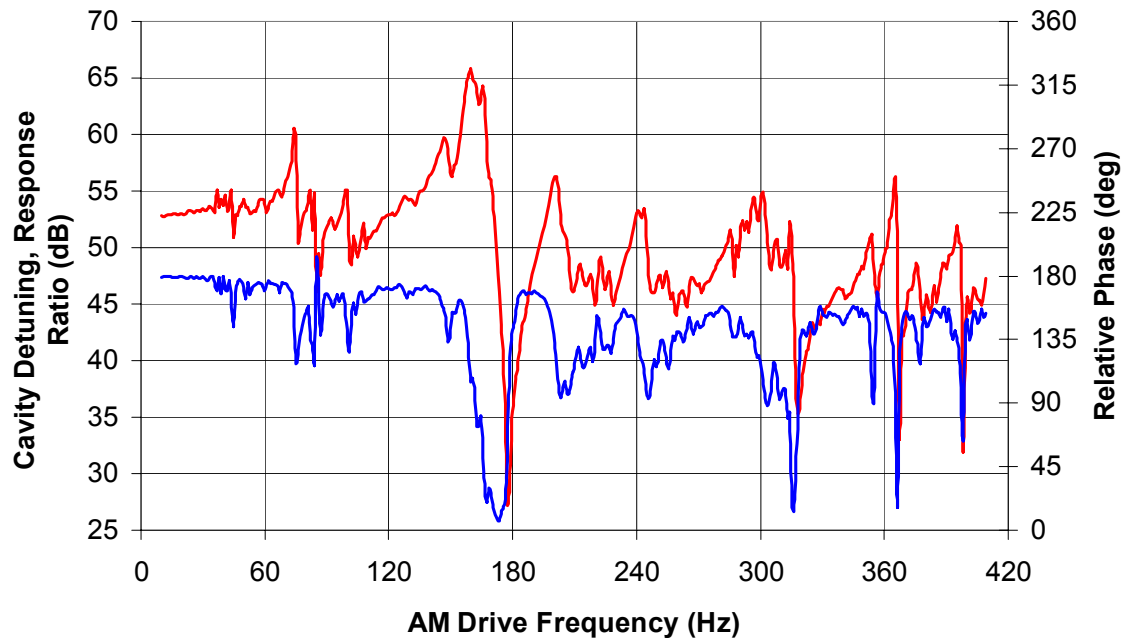
simple spectrum with few  
modes



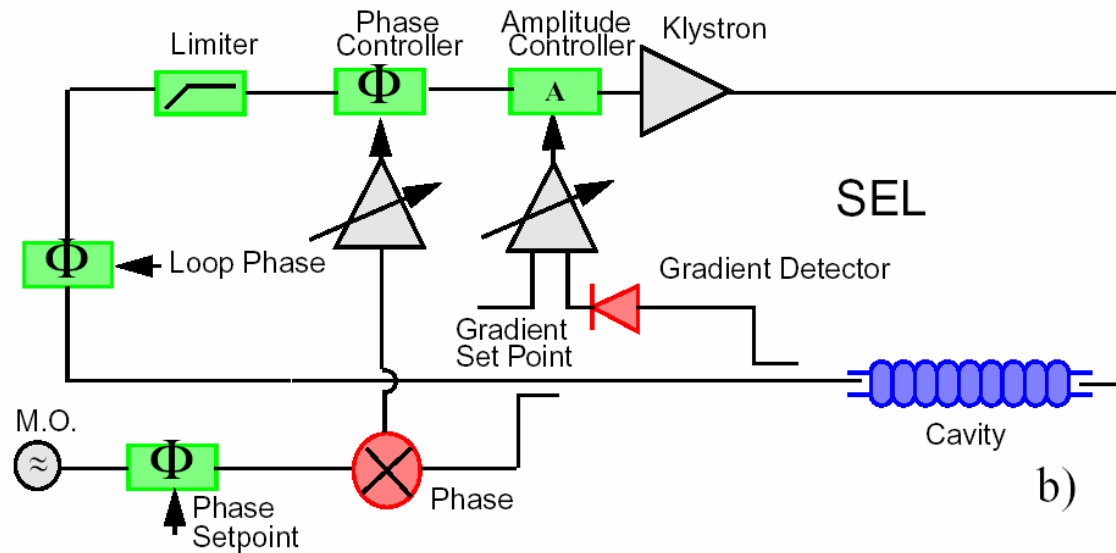
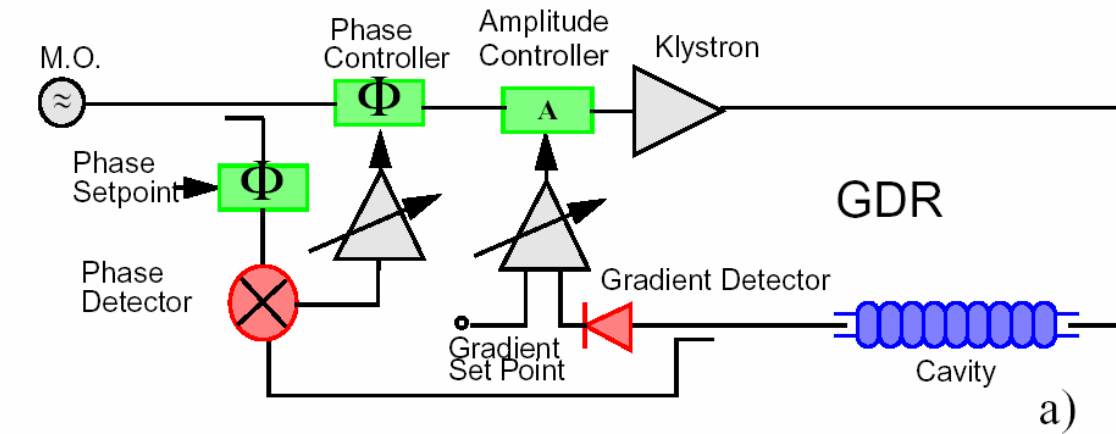
# Lorentz Transfer Function

TM-class cavities (Jlab, 6-cell elliptical, 805 MHz,  $\beta=0.61$ )  
Rich frequency spectrum from low to high frequencies  
Large variations between cavities

SNS Med  $\beta$  Cryomodule 3, Cavity Position 1, Lorentz Transfer Function  
(5MV/m CW)



# GDR and SEL



# Generator-Driven Resonator

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- In a generator-driven resonator the coupling between the electromagnetic and mechanical modes can lead to two ponderomotive instabilities
- **Monotonic instability** : Jump phenomenon where the amplitudes of the electromagnetic and mechanical modes increase or decrease exponentially until limited by non-linear effects
- **Oscillatory instability** : The amplitudes of both modes oscillate and increase at an exponential rate until limited by non-linear effects

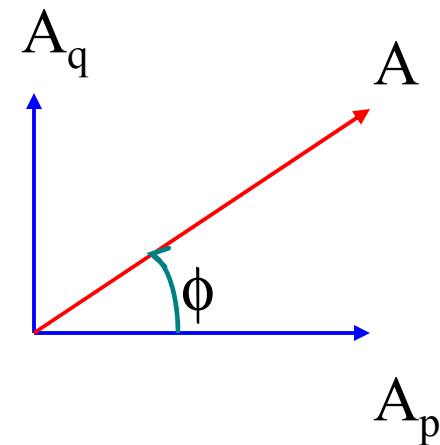
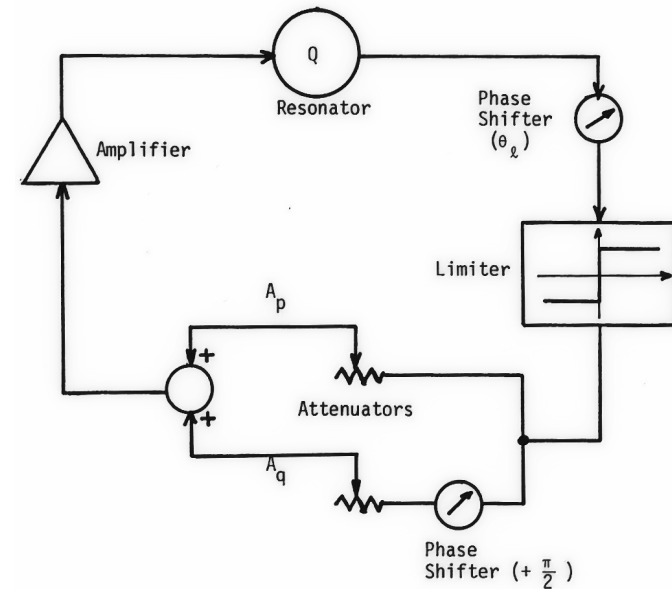
# Self-Excited Loop-Principle of Stabilization

Controlling the external phase shift  $\theta_l$  can compensate for the fluctuations in the cavity frequency  $\omega_c$  so the loop is phase locked to an external frequency reference  $\omega_r$ .

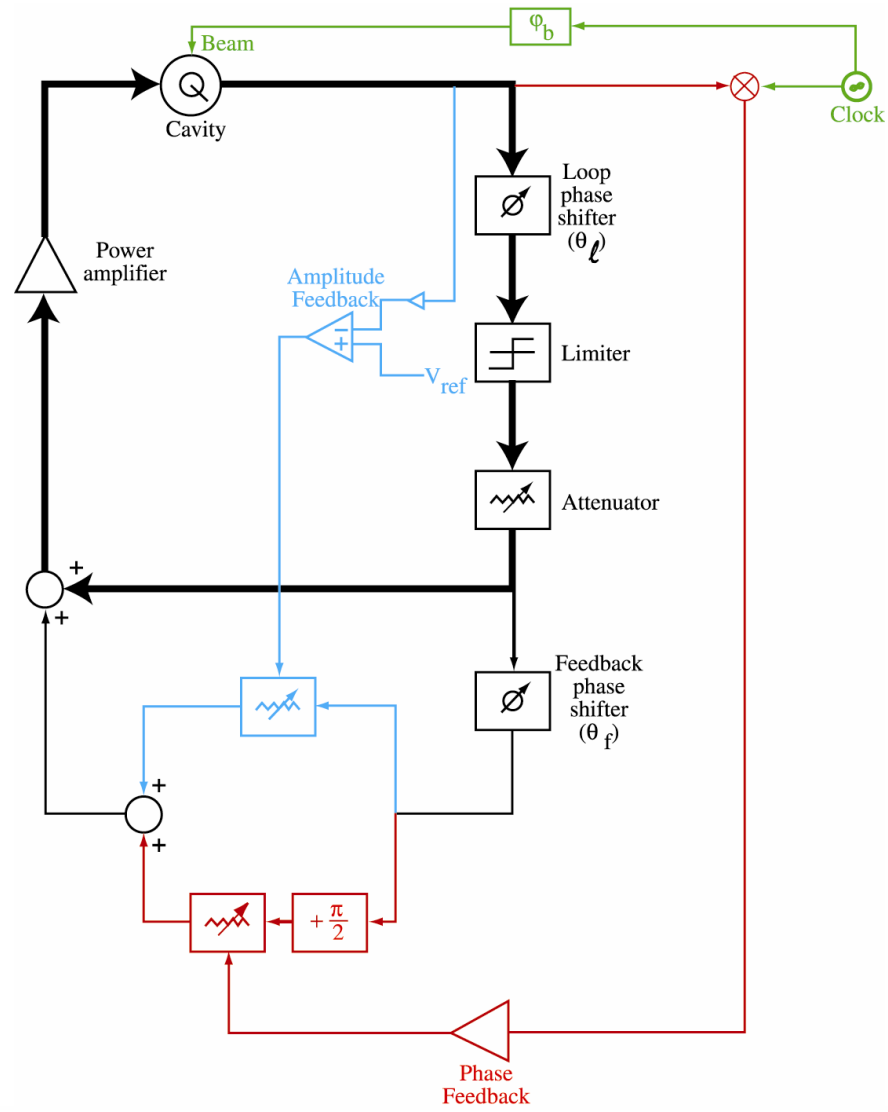
$$\omega = \omega_c + \frac{\omega_c}{2Q} \tan \theta_l$$

Instead of introducing an additional external controllable phase shifter, this is usually done by adding a signal in quadrature

→ The cavity field amplitude is unaffected by the phase stabilization even in the absence of amplitude feedback.



# Self-Excited Loop – Block Diagram





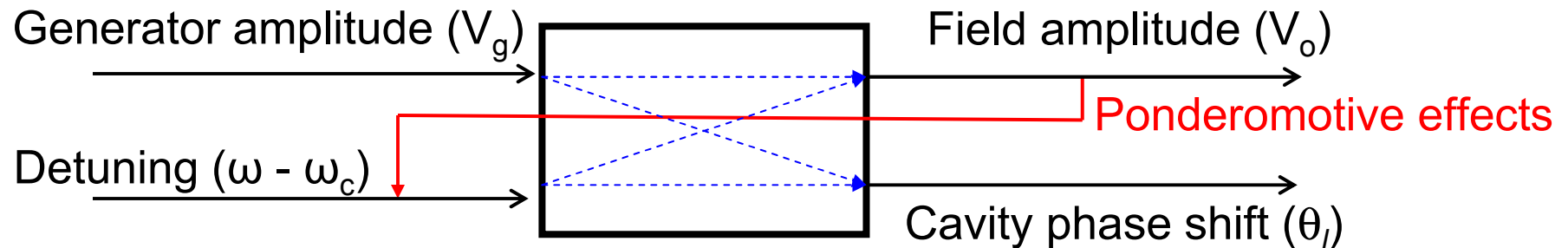
# Self-Excited Loop

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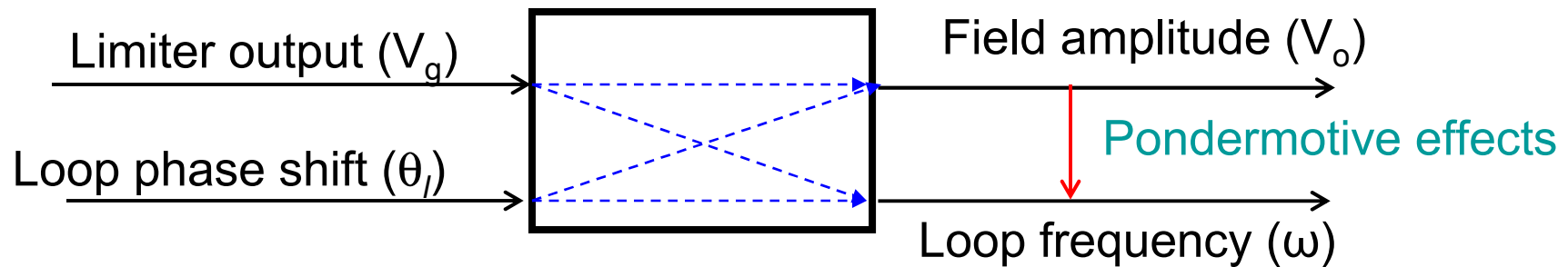
- **Resonators operated in self-excited loops in the absence of feedback are free of ponderomotive instabilities. An SEL is equivalent to the ideal VCO.**
  - **Amplitude is stable**
  - **Frequency of the loop tracks the frequency of the cavity**
- **Phase stabilization can reintroduce instabilities, but they are easily controlled with small amount of amplitude feedback**

# Input-Output Variables

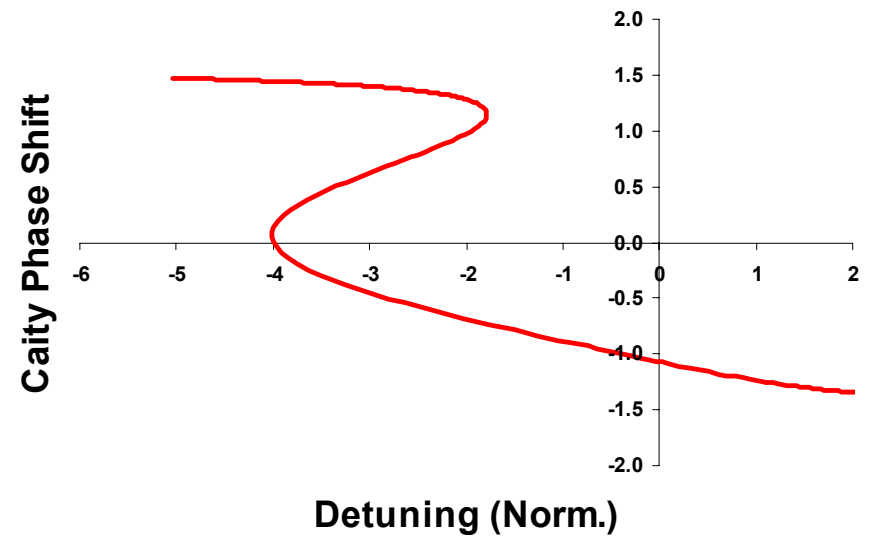
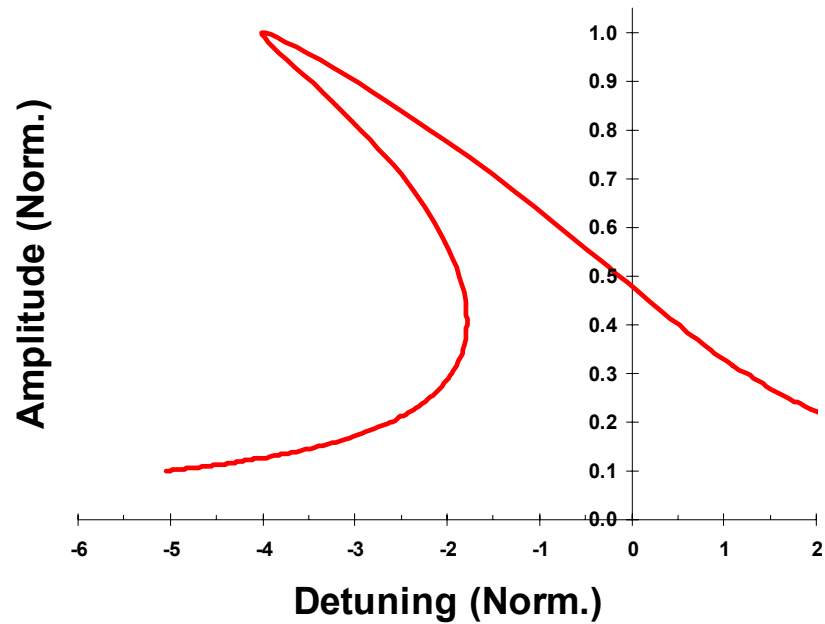
- **Generator - driven cavity**



- **Cavity in a self-excited loop**

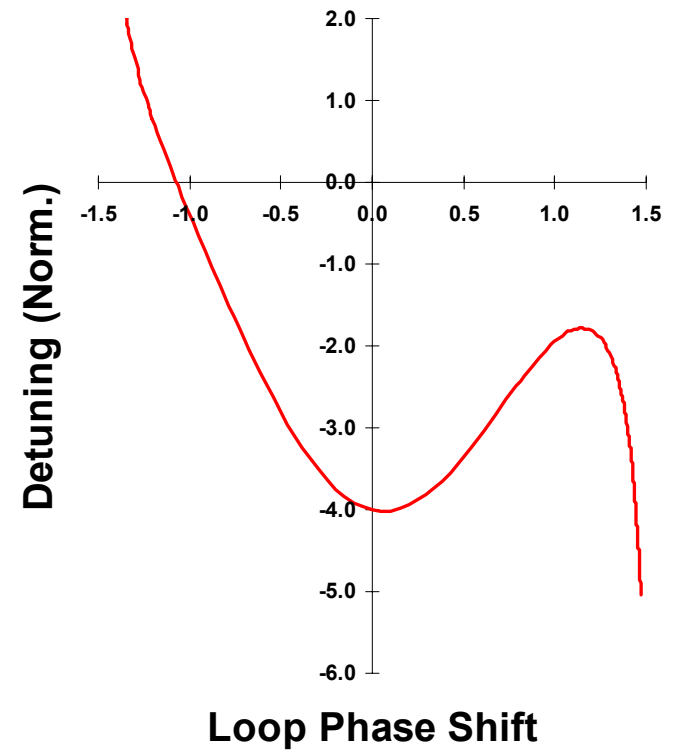
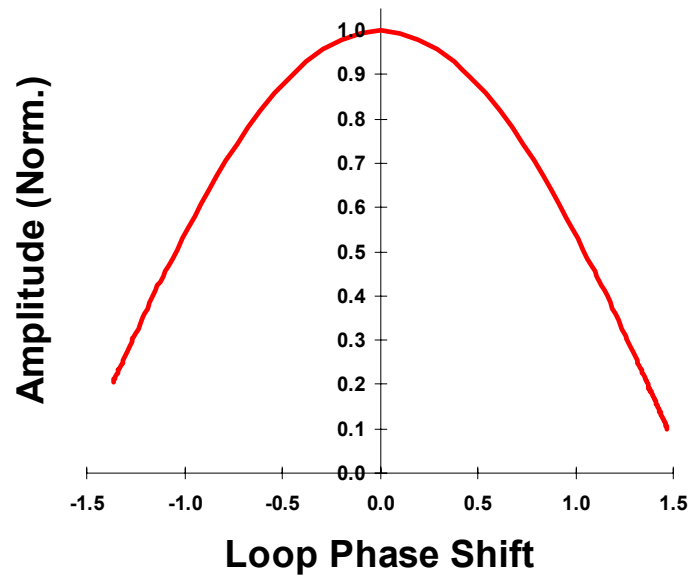


# Input-Output Variables Generator-Driven Resonator



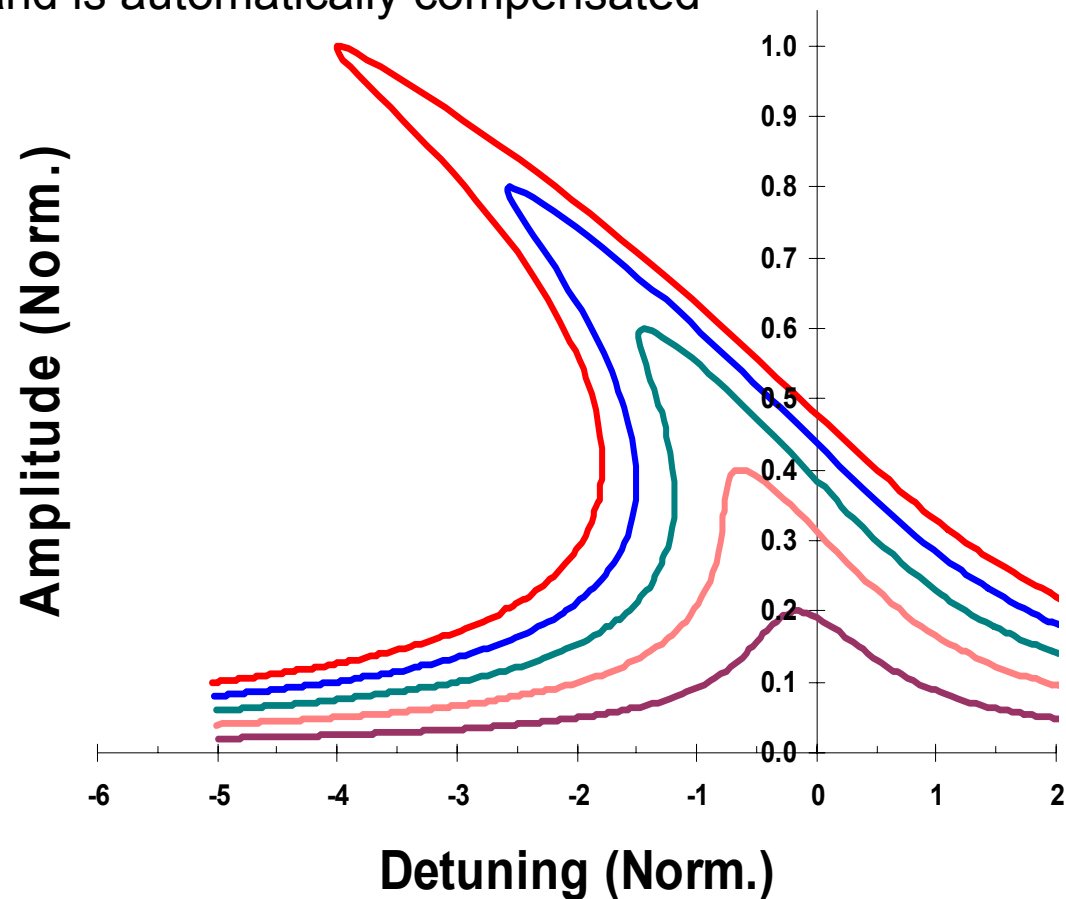
# Input-Output Variables

## Self-Excited Loop



# Lorentz Detuning

During transient operation (rise time and decay time) the loop frequency automatically tracks the resonator frequency. Lorentz detuning has no effect and is automatically compensated



# Microphonics

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- **Microphonics: changes in frequency caused by connections to the external world**
  - Vibrations
  - Pressure fluctuations

**When phase and amplitude feedbacks are active, ponderomotive effects can change the response to external disturbances**

$$\delta\ddot{\omega}_\mu + \frac{2}{\tau_\mu} \delta\dot{\omega}_\mu + \Omega_\mu^2 \delta\omega_\mu = -2\Omega_\mu^2 k_\mu V_o^2 \delta v + n(t)$$

# Microphonics

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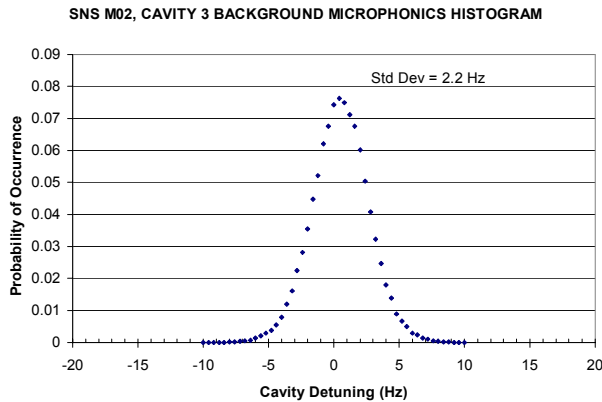
Two extreme classes of driving terms:

- Deterministic, monochromatic
  - Constant, well defined frequency
  - Constant amplitude
- Stochastic
  - Broadband (compared to bandwidth of mechanical mode)
  - Will be modeled by gaussian stationary white noise process

# Microphonics (probability density)

Single gaussian

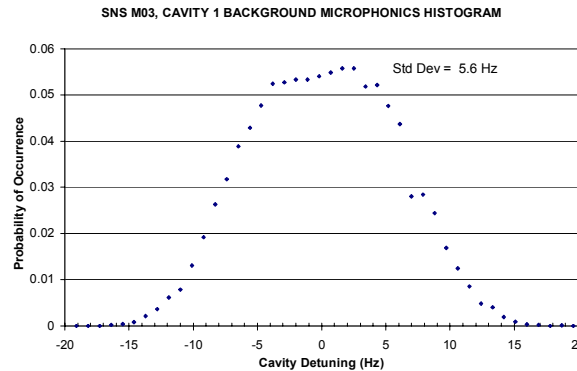
Noise driven



805 MHz TM

Bimodal

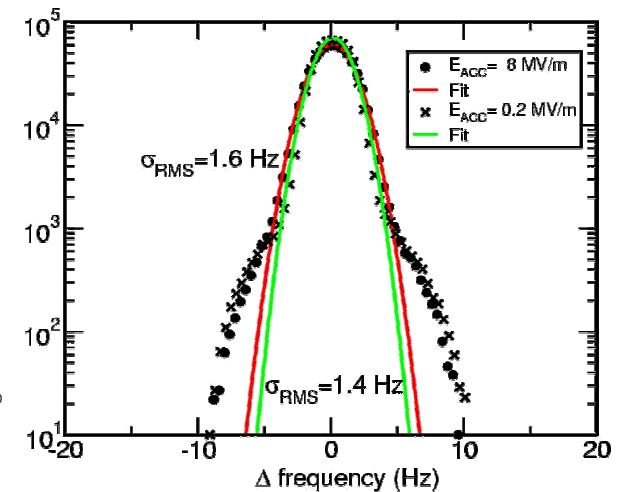
Single-frequency driven



805 MHz TM

Multi-gaussian

Non-stationary noise



172 MHz TEM

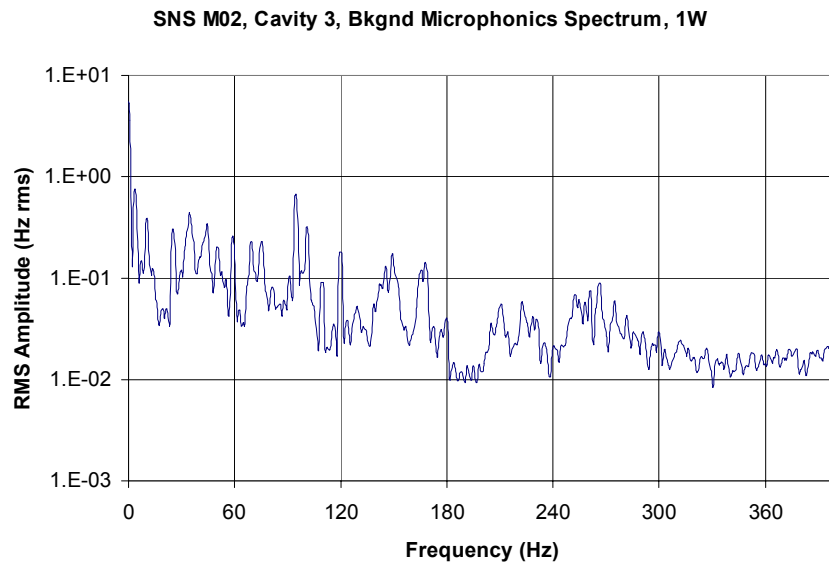


# Microphonics (frequency spectrum)

**TM-class cavities (JLab, 6-cell elliptical, 805 MHz,  $\beta=0.61$ )**

**Rich frequency spectrum from low to high frequencies**

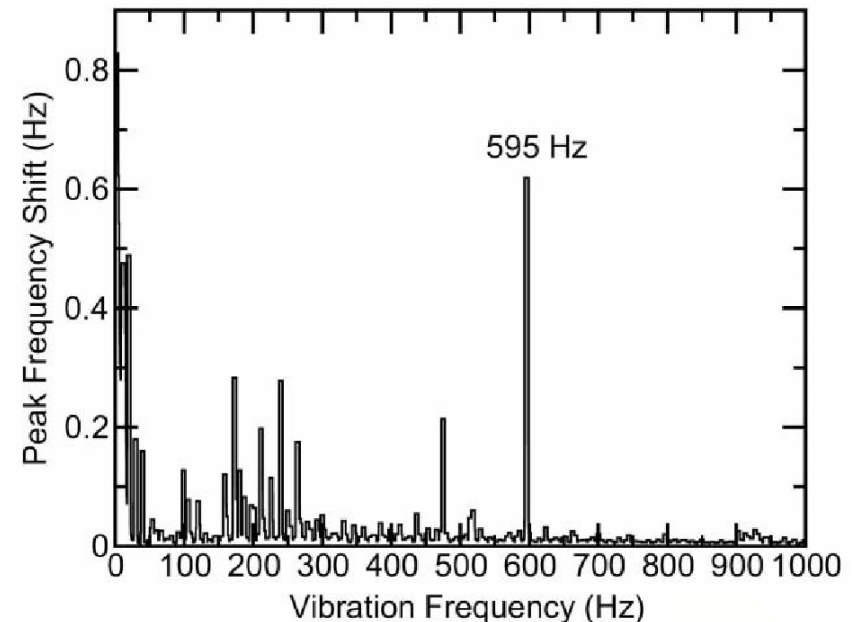
**Large variations between cavities**



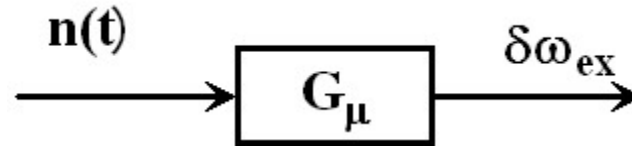
**TEM-class cavities (ANL, single-spoke, 354 MHz,  $\beta=0.4$ )**

**Dominated by low frequency (<10 Hz) from pressure fluctuations**

**Few high frequency mechanical modes that contribute little to microphonics level.**

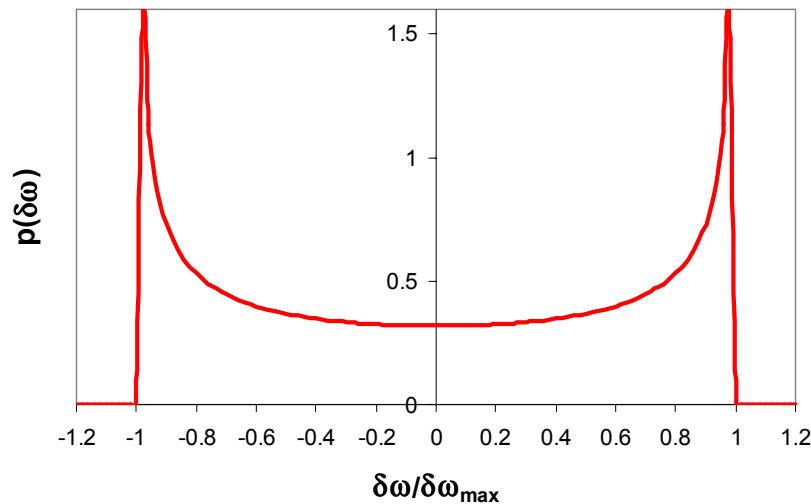


# Probability Density (histogram)



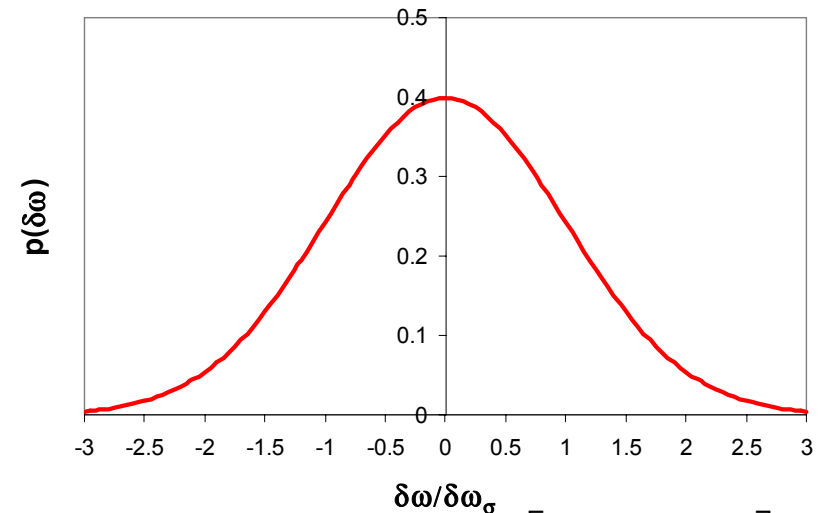
Harmonic oscillator ( $\Omega_\mu, \tau_\mu$ ) driven by:

Single frequency, constant amplitude



$$p(\delta\omega) = \frac{1}{\pi\sqrt{\delta\omega_{\max}^2 - \delta\omega^2}}$$

White noise, gaussian



$$p(\delta\omega) = \frac{1}{\sigma_\omega\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\delta\omega}{\sigma_\omega}\right)^2\right]$$

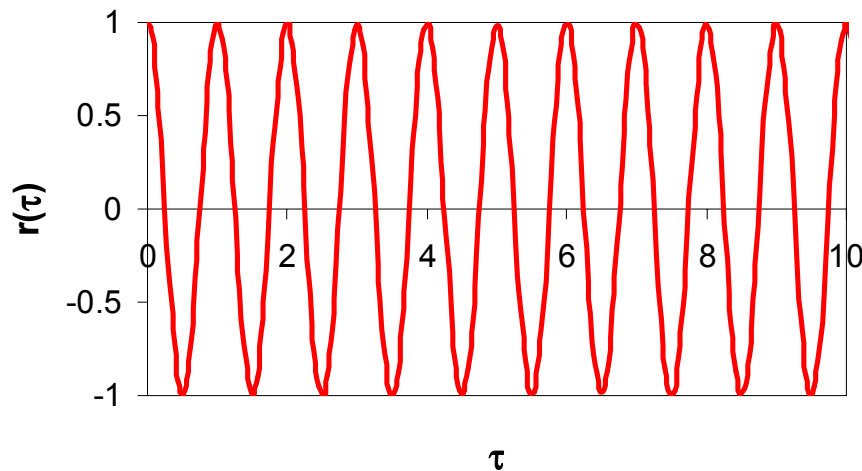
# Autocorrelation Function

$$R_x(\tau) = \langle x(t) x(t + \tau) \rangle = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T x(t) x(t + \tau) dt$$

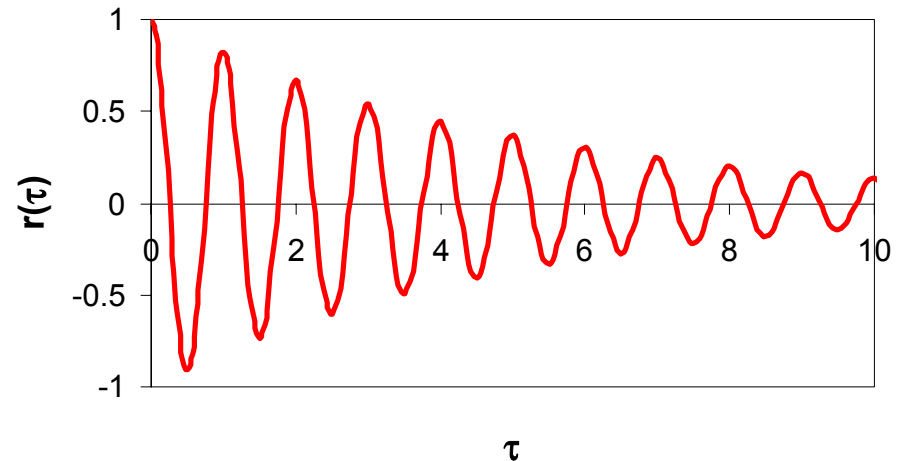
Harmonic oscillator ( $\Omega_\mu, \tau_\mu$ ) driven by:

Single frequency, constant amplitude

White noise, gaussian



$$r_{\delta\omega}(\tau) = \frac{R_{\delta\omega}(\tau)}{R_{\delta\omega}(0)} = \cos(\omega_d \tau)$$



$$r_{\delta\omega}(\tau) = \frac{R_{\delta\omega}(\tau)}{R_{\delta\omega}(0)} = \cos(\Omega_\mu \tau) e^{-|\tau/\tau_\mu|}$$

# Stationary Stochastic Processes

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$x(t)$ : stationary random variable

Autocorrelation function:  $R_x(\tau) = \langle x(t)x(t+\tau) \rangle = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T x(t)x(t+\tau) dt$

Spectral Density  $S_x(\omega)$ : Amount of power between  $\omega$  and  $d\omega$

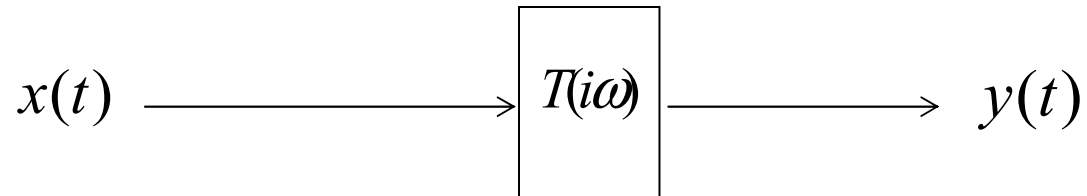
$S_x(\omega)$  and  $R_x(\tau)$  are related through the Fourier Transform (Wiener-Khintchine)

$$S_x(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R_x(\tau) e^{-i\omega\tau} d\tau \qquad R_x(\tau) = \int_{-\infty}^{\infty} S_x(\omega) e^{i\omega\tau} d\omega$$

Mean square value:  $\langle x^2 \rangle = R_x(0) = \int_{-\infty}^{\infty} S_x(\omega) d\omega$

# Stationary Stochastic Processes

For a stationary random process driving a linear system



$$\langle y^2 \rangle = R_y(0) = \int_{-\infty}^{+\infty} S_y(\omega) d\omega \quad \langle x^2 \rangle = R_x(0) = \int_{-\infty}^{+\infty} S_x(\omega) d\omega$$

$R_y(\tau)$  [ $R_x(\tau)$ ]: auto correlation function of  $y(t)$  [ $x(t)$ ]

$S_y(\omega)$  [ $S_x(\omega)$ ]: spectral density of  $y(t)$  [ $x(t)$ ]

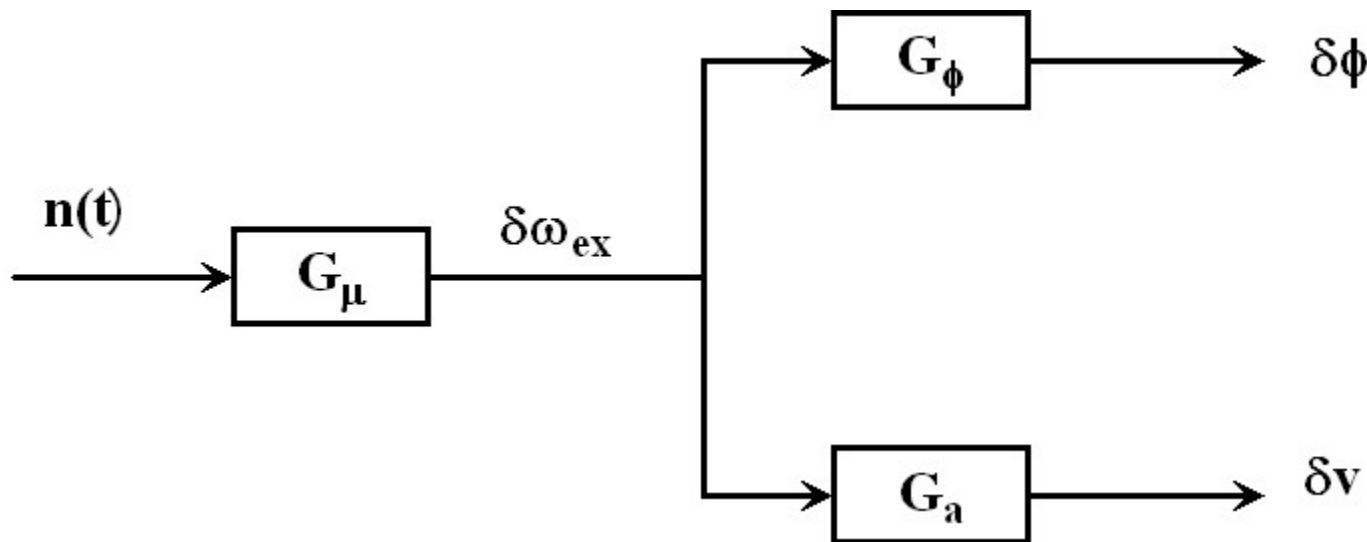
$$S_y(\omega) = S_x(\omega) |T(i\omega)|^2$$

$$\langle y^2 \rangle = \int_{-\infty}^{+\infty} S_x(\omega) |T(i\omega)|^2 d\omega$$

# Performance of Control System

Residual phase and amplitude errors caused by microphonics  
Can also be done for beam current amplitude and phase  
fluctuations

Assume a single mechanical oscillator of frequency  $\Omega_\mu$  and decay time  $\tau_\mu$   
excited by white noise of spectral density  $A^2$



# Performance of Control System

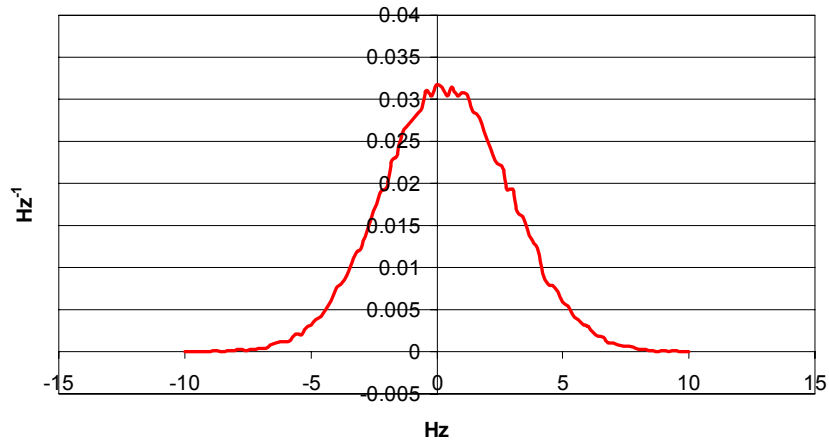
$$\langle \delta\omega_{ex}^2 \rangle = A^2 \int_{-\infty}^{+\infty} |G_{\mu}(i\omega)|^2 d\omega = A^2 \int_{-\infty}^{+\infty} \frac{d\omega}{\left| -\omega^2 + \frac{2}{\tau_{\mu}} i\omega + \Omega_{\mu}^2 \right|^2} = A^2 \frac{\pi\tau_{\mu}}{2\Omega_{\mu}^2}$$

$$\langle \delta v^2 \rangle = A^2 \int_{-\infty}^{+\infty} |G_{\mu}(i\omega) G_a(i\omega)|^2 d\omega = \langle \delta\omega_{ex}^2 \rangle \frac{2\Omega_{\mu}^2}{\pi\tau_{\mu}} \int_{-\infty}^{+\infty} \left| \frac{G_a(i\omega)}{-\omega^2 + \frac{2}{\tau_{\mu}} i\omega + \Omega_{\mu}^2} \right|^2 d\omega$$

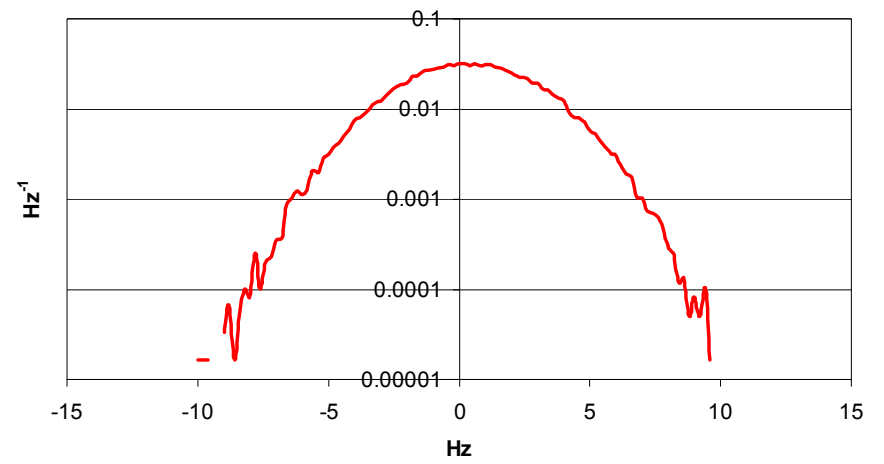
$$\langle \delta\phi^2 \rangle = A^2 \int_{-\infty}^{+\infty} |G_{\mu}(i\omega) G_{\phi}(i\omega)|^2 d\omega = \langle \delta\omega_{ex}^2 \rangle \frac{2\Omega_{\mu}^2}{\pi\tau_{\mu}} \int_{-\infty}^{+\infty} \left| \frac{G_{\phi}(i\omega)}{-\omega^2 + \frac{2}{\tau_{\mu}} i\omega + \Omega_{\mu}^2} \right|^2 d\omega$$

# The Real World

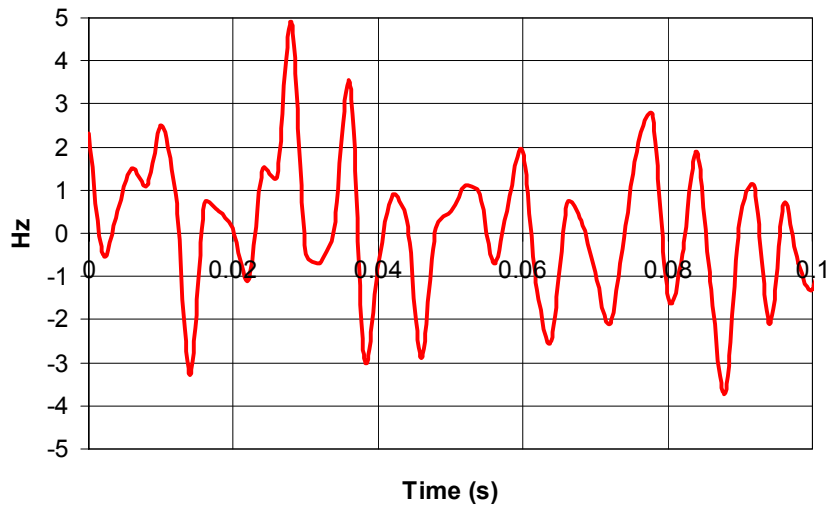
Probability Density



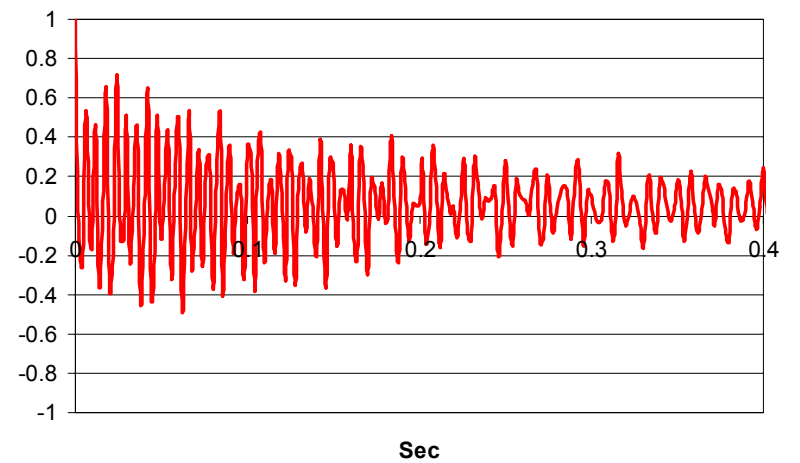
Probability Density



Microphonics



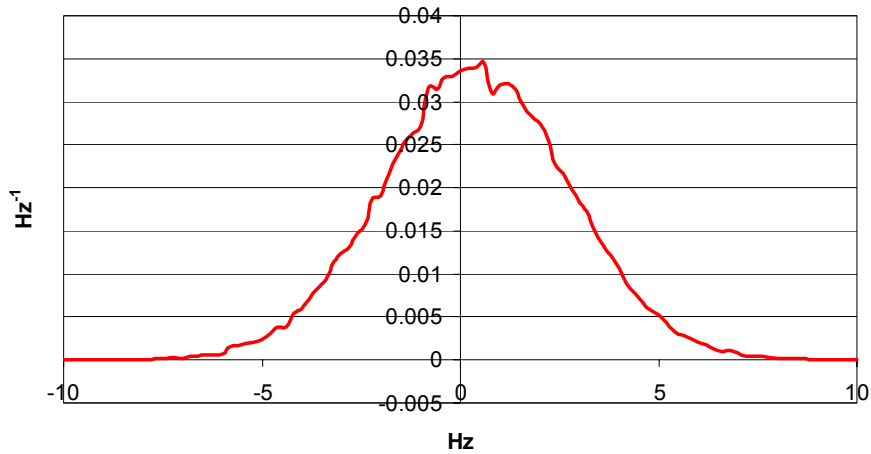
Normalized Autocorrelation Function



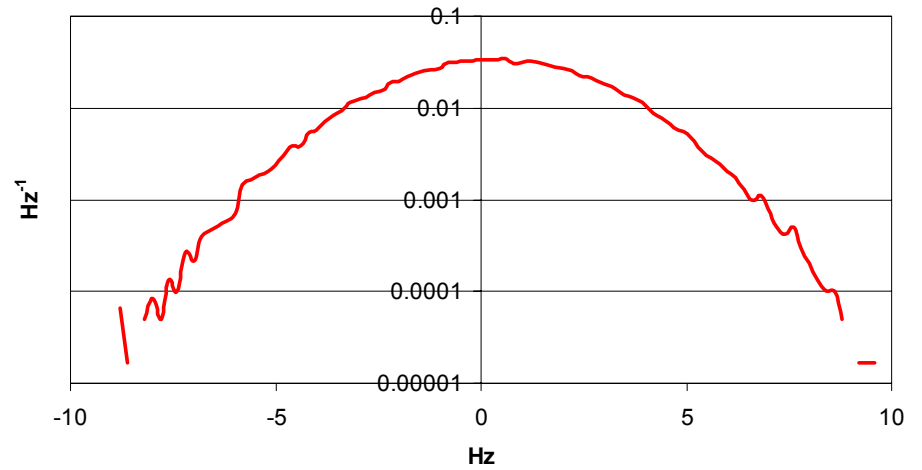


# The Real World

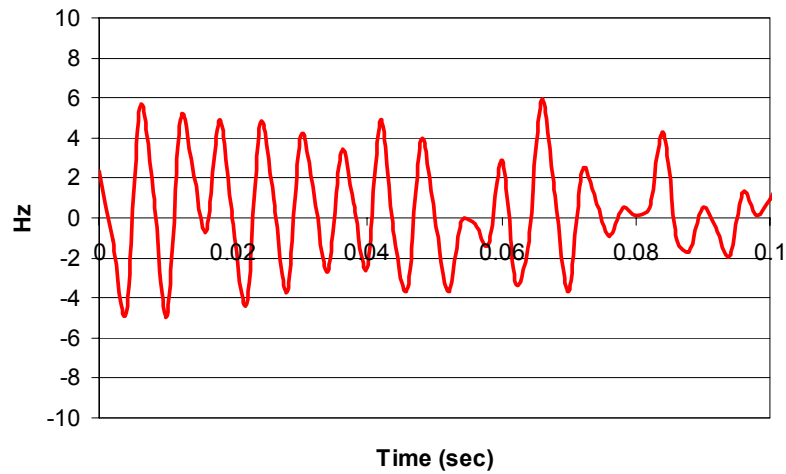
Probability Density



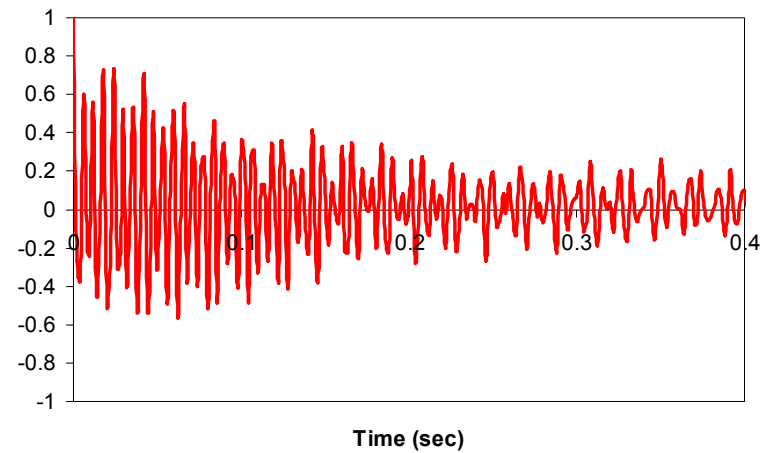
Probability Density



Microphonics

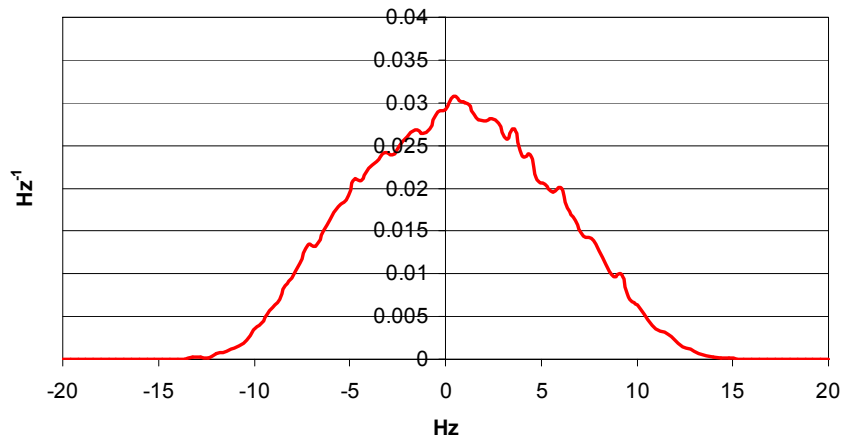


Normalized Autocorrelation Function

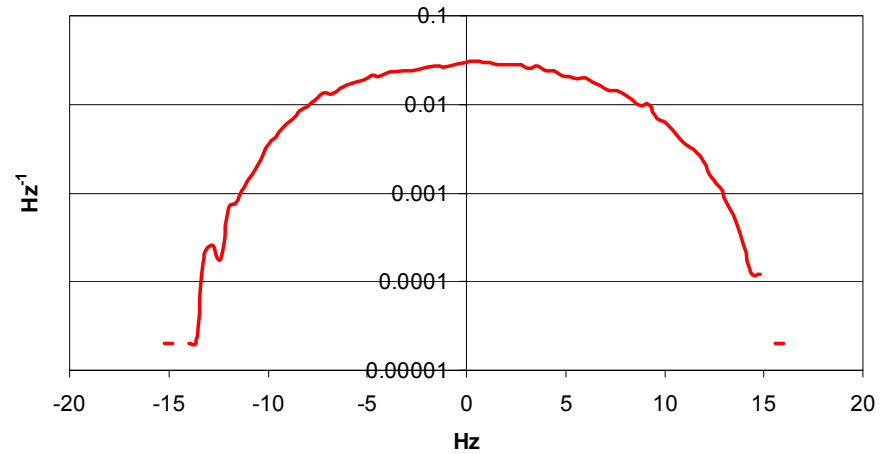


# The Real World

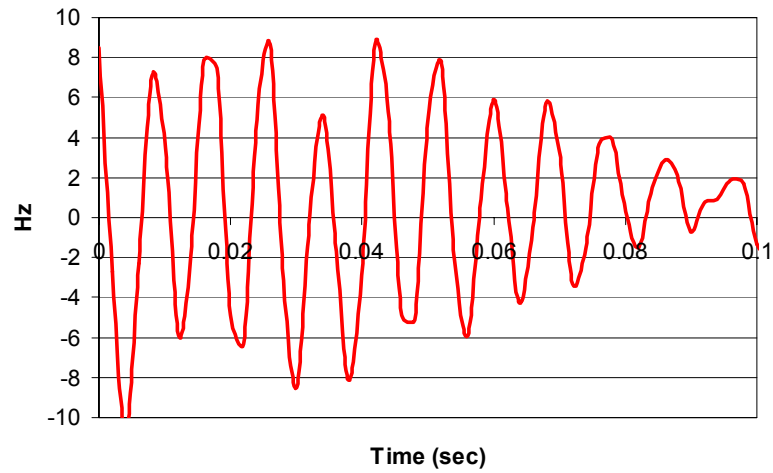
Probability density



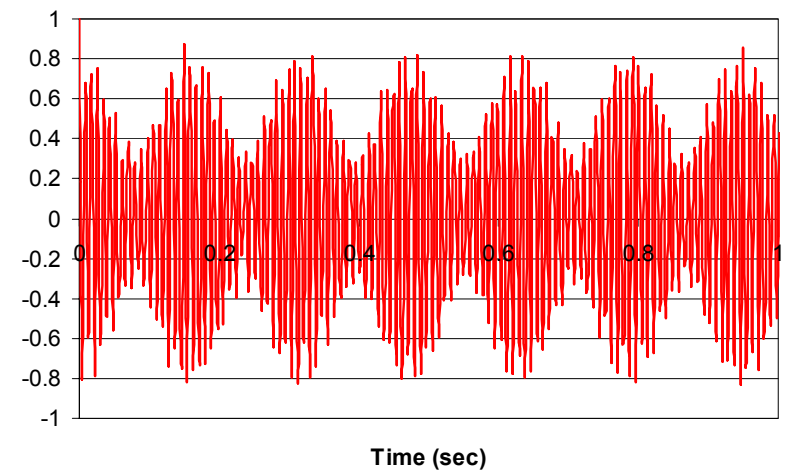
Probability density



Microphonics



Normalized Autocorrelation Function



# Piezo control of microphonics

MSU, 6-cell elliptical 805 MHz,  $\beta=0.49$

Adaptive feedforward compensation

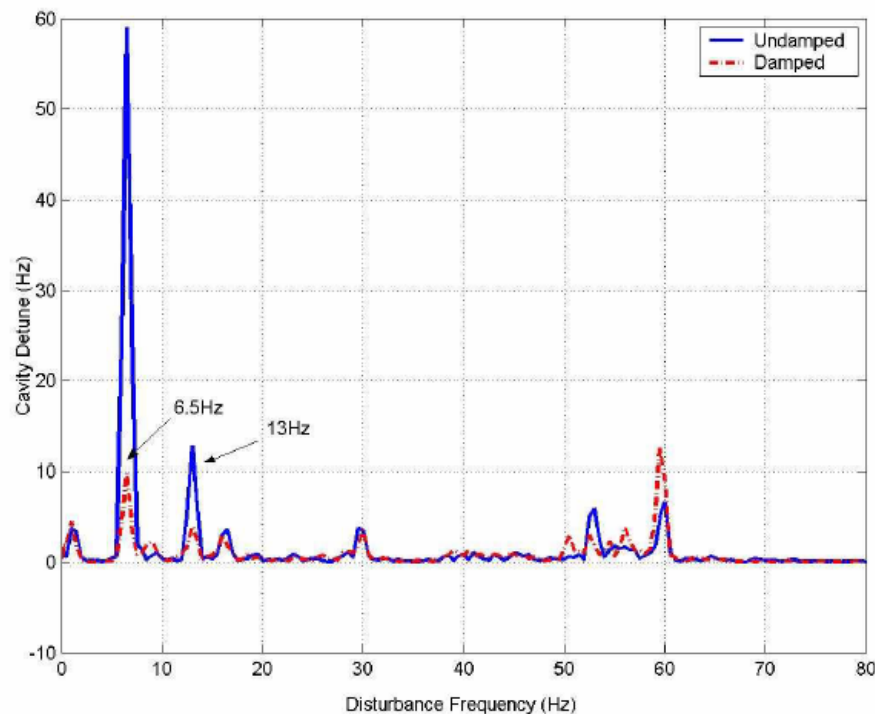


Figure 2. Active damping of helium oscillations at 2K.

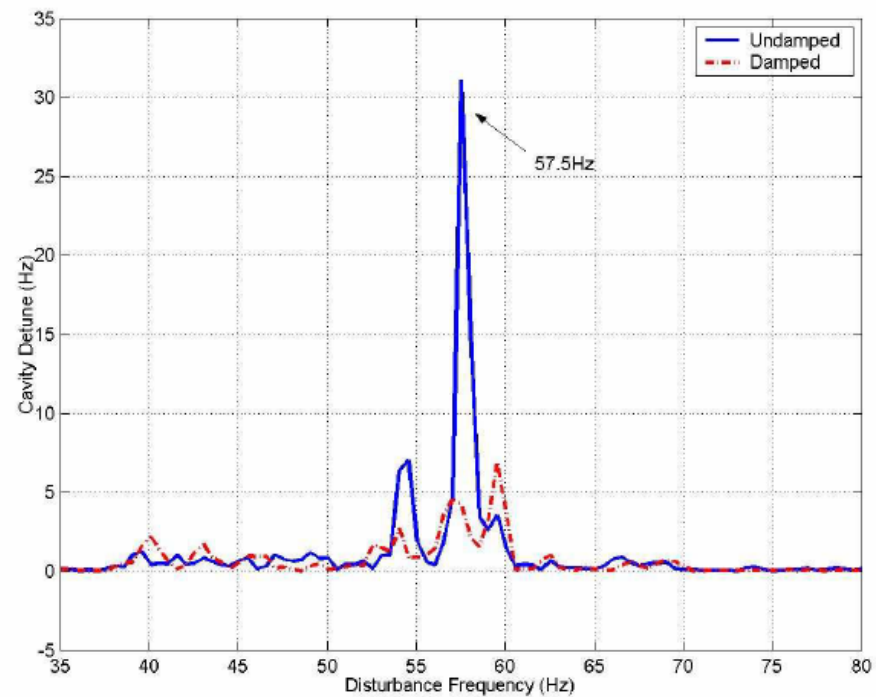
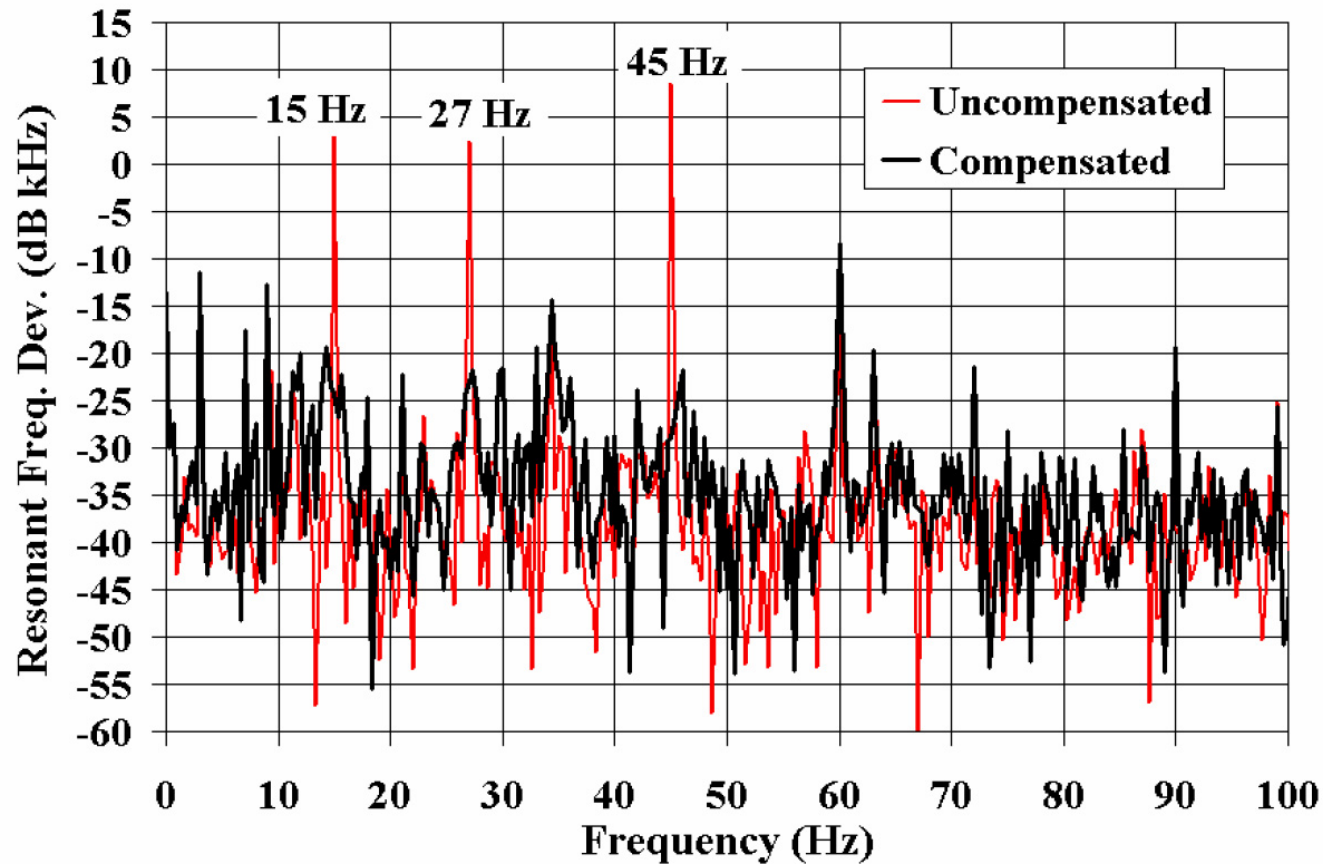


Figure 3. Active damping of external vibration at 2K.

# Piezo Control of Microphonics

FNAL, 3-cell 3.9 GHz



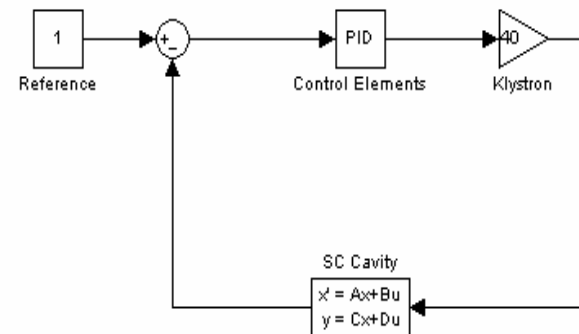
# SEL and GDR

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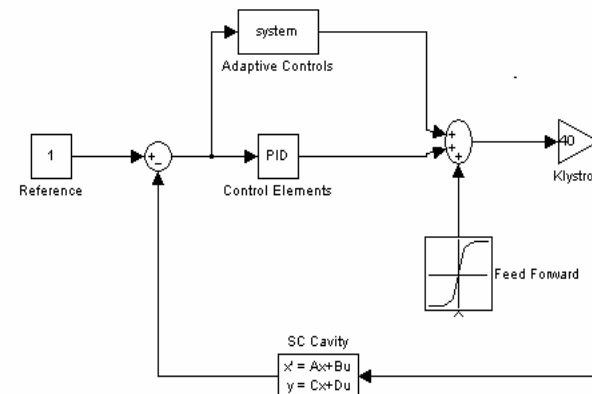
- **SEL are best suited for high gradient, high-loaded Q cavities operated cw.**
  - Well behaved with respect to ponderomotive instabilities
  - Unaffected by Lorentz detuning at power up
  - Able to run independently of external rf source
  - Rise time can be random and slow (starts from noise)
  
- **GDR are best suited for low-Q cavities operated for short pulse length.**
  - Fast predictable rise time
  - Power up can be hampered by Lorentz detuning

# SC Control Systems

- CW accelerators (Atlas, CEBAF) use simple proportional negative feedback.



- Pulsed accelerators (TESLA, SNS) need more complex control methods, adaptive control, and feed forward techniques.



# Control System Example

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At CEBAF, Nuclear experiments require an energy spread of  $\sim 10^{-4}$

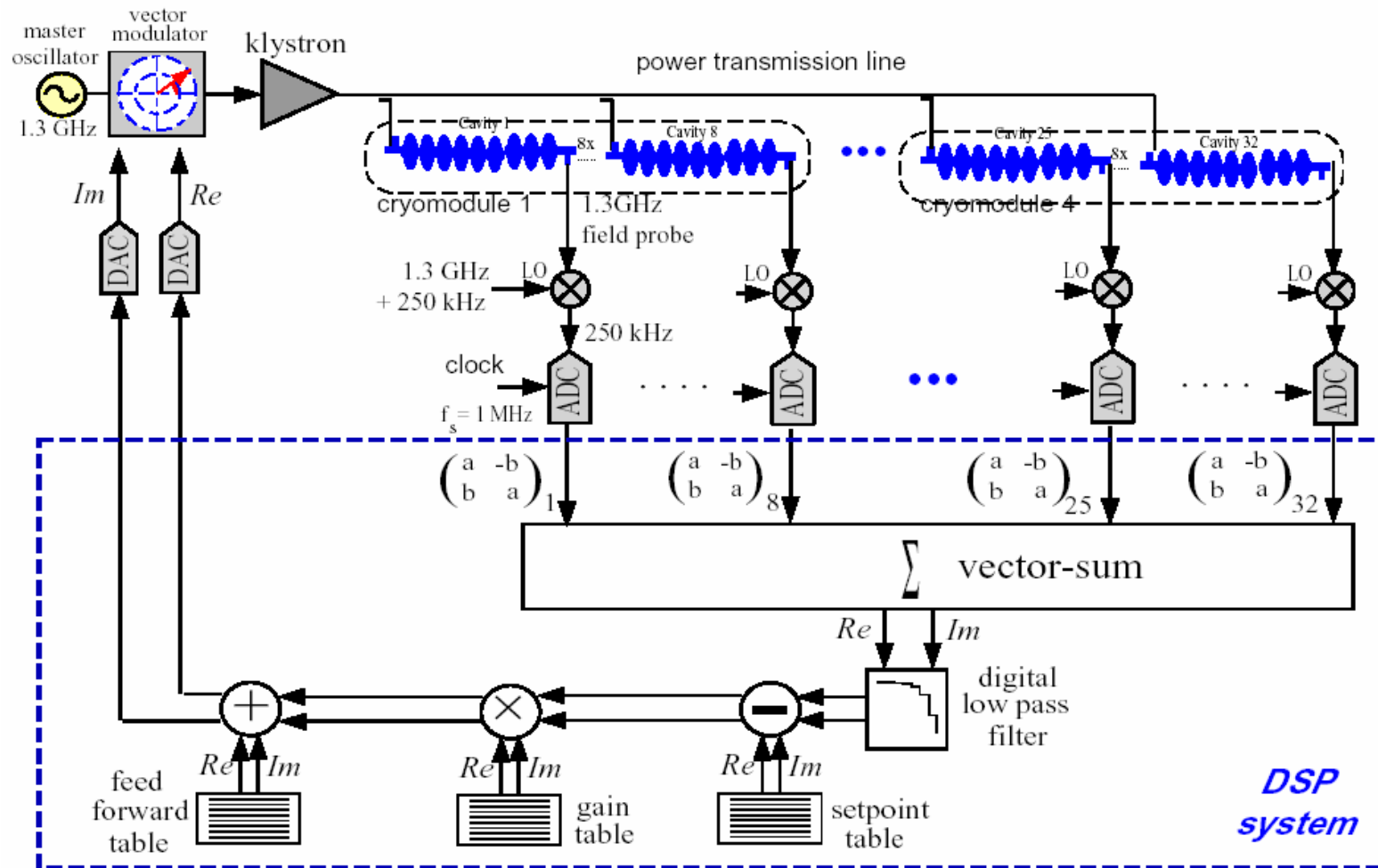
To meet this each individual cavity must have no more than  $\sim 10^{-5}$  amplitude variation.

$$[\Delta E/E \sim 1/N^{1/2} \text{ where } N \text{ is the number of cavities}]$$

Background microphonics are 5% (peak) do to  $Q_L = 10^7$

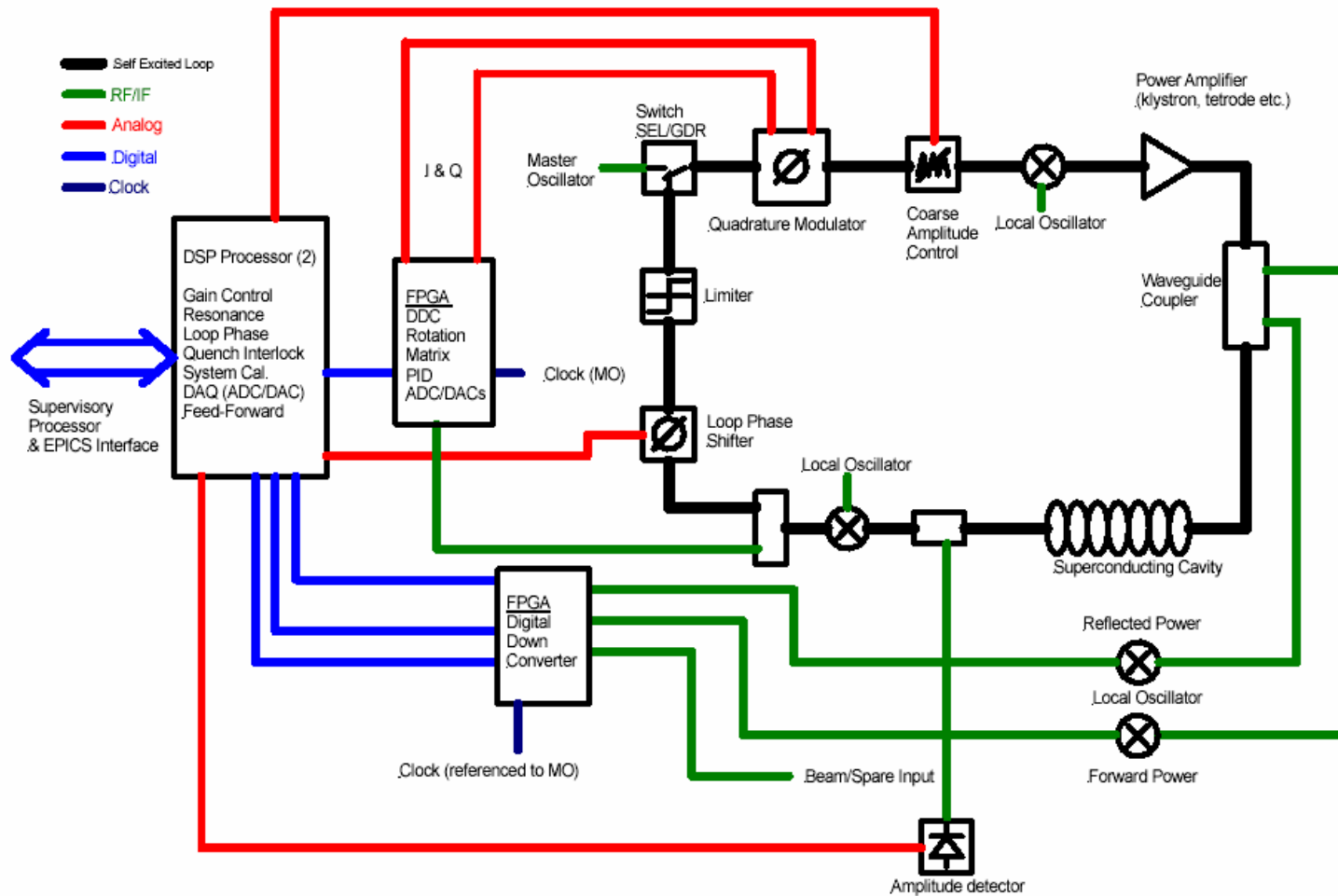
Therefore gain required to control the cavity field is 500 or  $\sim 53$  dB in gain.

# TESLA Control System

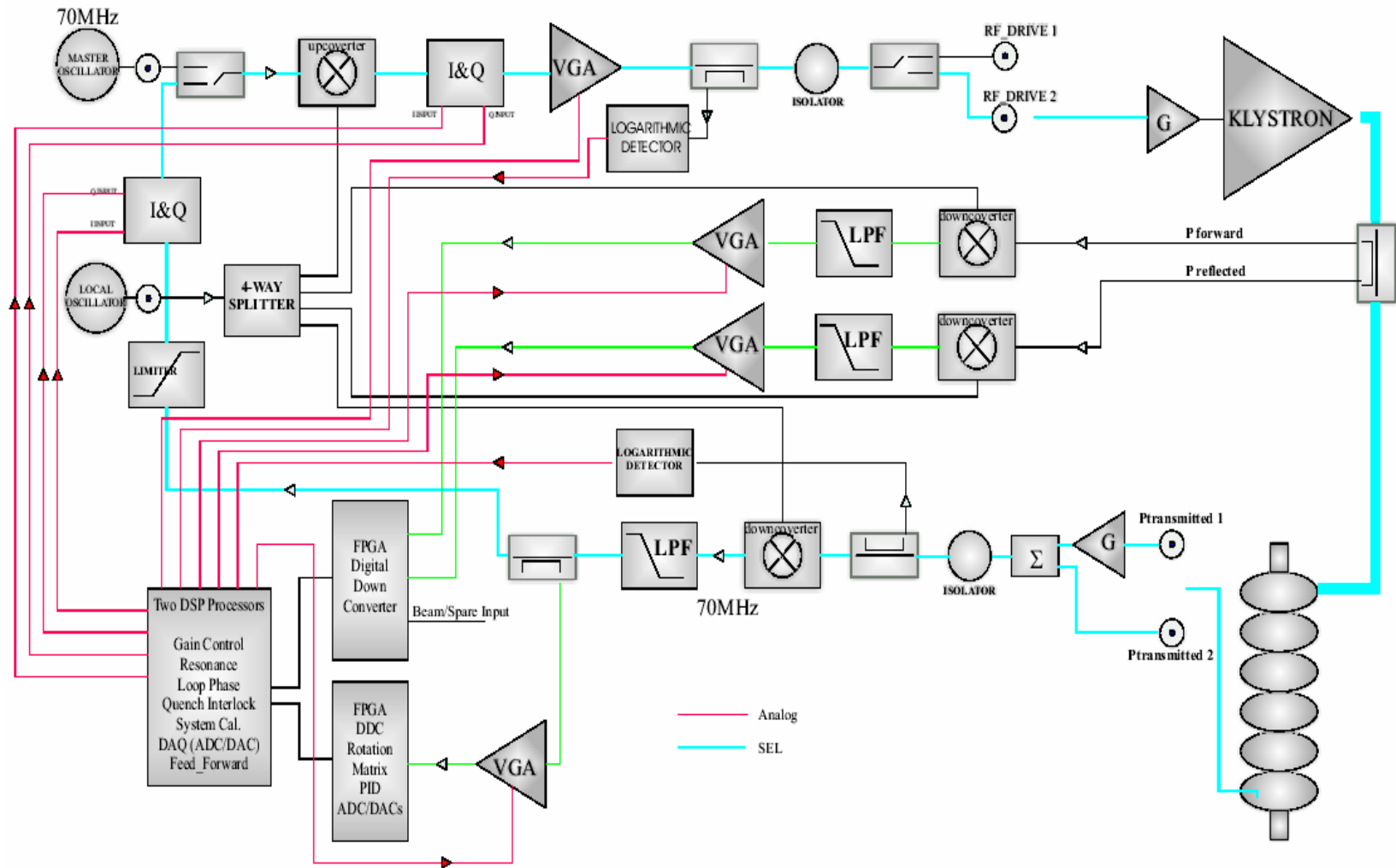




# Basic LLRF Block Diagram



# Low level rf control development



Concept for a LLRF control system

# Pulsed Operation

- Under pulsed operation Lorentz detuning can have a complicated dynamic behavior

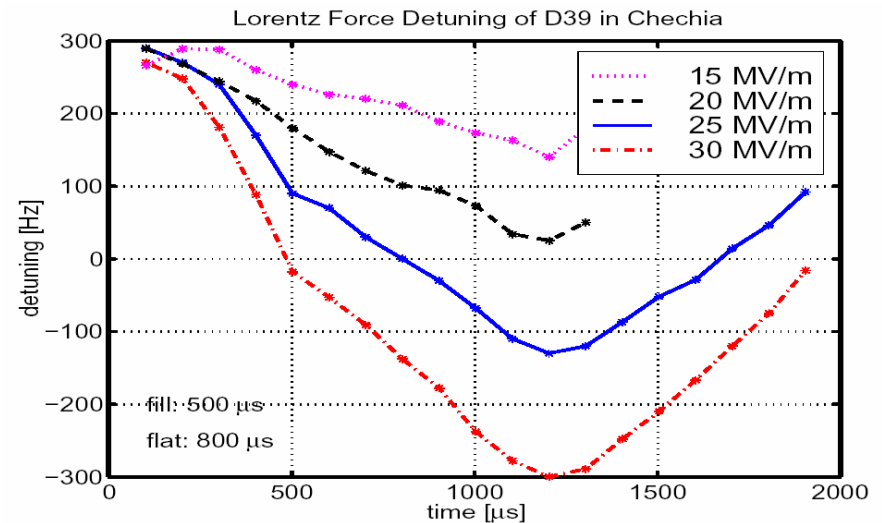
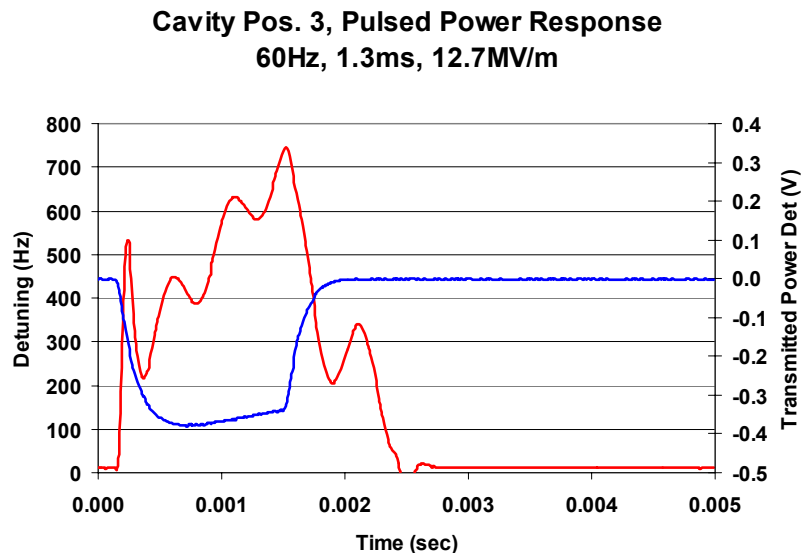


Fig. 2: Lorentz force detuning measured for a TESLA cavity at different gradients.

# Pulsed Operation

- Fast piezoelectric tuners can be used to compensate the dynamic Lorentz detuning

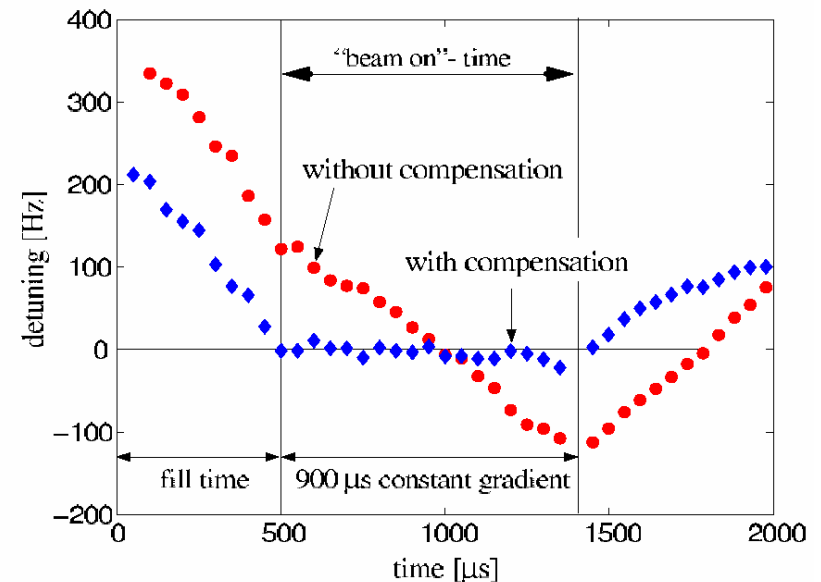
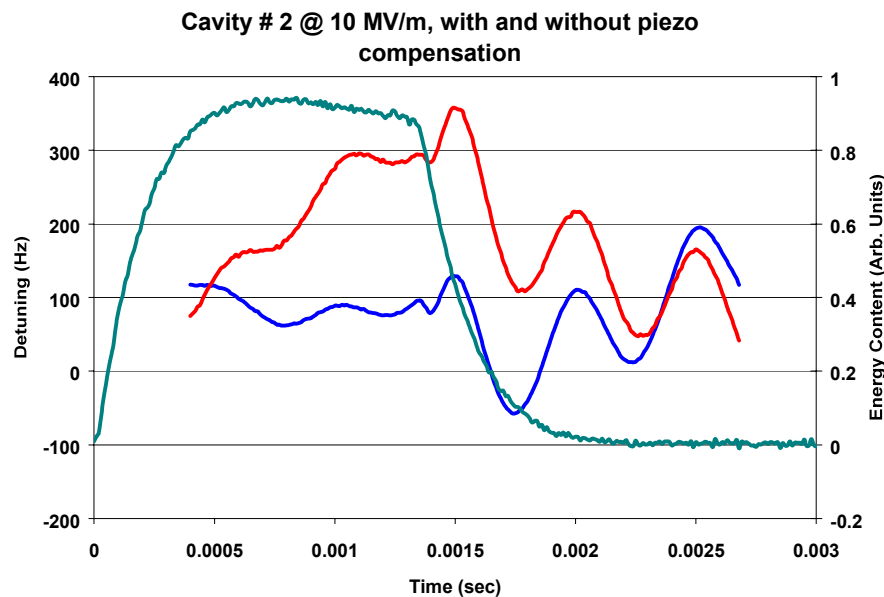


Figure 2. Lorentz force compensation at the TTF

# Status of Microphonics Control

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- Microphonics and ponderomotive instabilities issues in high-Q SRF cavities were “hot topics” in the early days (~70s), especially in low- $\beta$  applications
- They were solved and are well understood
- They are being rediscovered in medium- to high- $\beta$  applications
- Today’s challenges:
  - Large scale (cavities and accelerators): need for optimization
  - Finite beam loading
    - Small but non-negligible current (e.g. RIA)
    - Low current resulting from the not quite perfect cancellation of 2 large currents (ERLs)